Math 126: Practice Midterm Exam
Fall 2015

These questions are taken (largely) from past exams. They are meant to represent the types of questions that you will see on the midterm exam, but are not meant to be a comprehensive list. Look to past quizzes, past homework assignments, and class notes for other study resources in addition to these exercises.

1. Tell whether each series converges conditionally, converges absolutely, or diverges. Be sure to use an appropriate test to justify your answers.

   (a)
   $$\sum_{n=0}^{\infty} \frac{\ln n}{2n+1}$$

   (b)
   $$\sum_{n=0}^{\infty} \frac{2n+1}{\sqrt{3n^2+2}}$$

   (c)
   $$\sum_{n=0}^{\infty} \left(\frac{2n+1}{4n-2}\right)^n$$

   (d)
   $$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2+1}}$$

   (e)
   $$\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^3}$$

   (f)
   $$\sum_{n=1}^{\infty} \frac{\sin(n)}{\sqrt{n^3+1}}$$

2. Is $f(x) = \sin(x^2)$ an even or an odd function? What about $f(x) = \sin(x^3)$. Use Taylor Series to justify your answers.

3. Calculate the following sums.

(a) $4+\frac{4}{3}+\frac{4}{9}+\ldots$

(b) $1-1+\frac{1}{2}-\frac{1}{6}+\frac{1}{24}-\ldots$

(c) $1-\frac{\pi^2}{2}+\frac{\pi^4}{24}-\frac{\pi^6}{720}+\ldots$

4. For which values of x does the series

$$\sum_{n=1}^{\infty} \frac{(2x+3)^n}{\sqrt{n+1}}$$

converge? Be sure to check endpoints.

5. Determine a series for $f(x) = e^{x^2}$ and use your series to determine

$$\lim_{x \to 0} \frac{e^{x^2} - 1 - x^2 - \frac{x^4}{2}}{x^6}$$

6. Find

$$\frac{d}{dx} \int_{2x}^{3x} \frac{1}{\sqrt{1+t^2}}\, dt$$

7. Find the following integrals and antiderivatives:

(a)

$$\int_{-2}^{2} (x+1)(x^2+1)\, dx$$

(b)

$$\int \cos^2(3x)\, dx$$

(c)

$$\int \frac{1}{1+x}\, dx$$

(d)

$$\int \frac{\ln(2x+4)}{x+2}\, dx$$

8. Tell whether each series converges conditionally, converges absolutely, or diverges. Be sure to use an appropriate test to justify your answers.

(a)

$$\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$$

(b)

$$\sum_{n=0}^{\infty} \frac{(-1)^n(n^3+1)}{\sqrt{n^7+2}}$$

(c)

$$\sum_{n=0}^{\infty} \frac{3^{2n}}{2^{3n}}$$

(d)

$$\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$$

9. Find the radius and interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n^2 2^n}.$$

10. Calculate the following sums.

(a) $2 - \frac{3}{2} + \frac{9}{8} - \frac{27}{32} + \ldots$

(b) $\frac{\pi}{3} - \frac{\pi^3}{3^3 3!} + \frac{\pi^5}{3^5 5!} - \frac{\pi^7}{3^7 7!} + \ldots$

11. (a) Determine a series for $f(x) = \cos(x^3)$.

(b) Is $f(x) = \cos(x^3)$ an even function or an odd function? How do you know?

(c) Find the *twelfth* derivative of $f(x) = \cos(x^3)$ evaluated at $x = 0$ (Leave your answer in terms of factorials).

(d) Find the *thirteenth* derivative of $f(x) = \cos(x^3)$ evaluated at $x = 0$.

12. Find

$$\frac{d}{dx} \int_5^{x^2+x} \ln(\sqrt{t+1})\, dt.$$

13. Find the following antiderivatives and integrals.

   (a) $\int \frac{x^3+1}{x^2}\, dx$

   (b) $\int_0^1 x e^{x^2}\, dx$

   (c) $\int \tan(3x)\, dx$

14. Find the following indefinite integral via u-substitution two different ways:

$$\int \sin(x)\cos(x)\, dx$$

   (a) By letting $u = \sin(x)$

   (b) By letting $u = \cos(x)$.

   Explain why your two answers are the same, even though they look different.

15. Suppose that $f(x)$ is a function that is both even and odd. Prove that $f(x) = 0$ for all x.