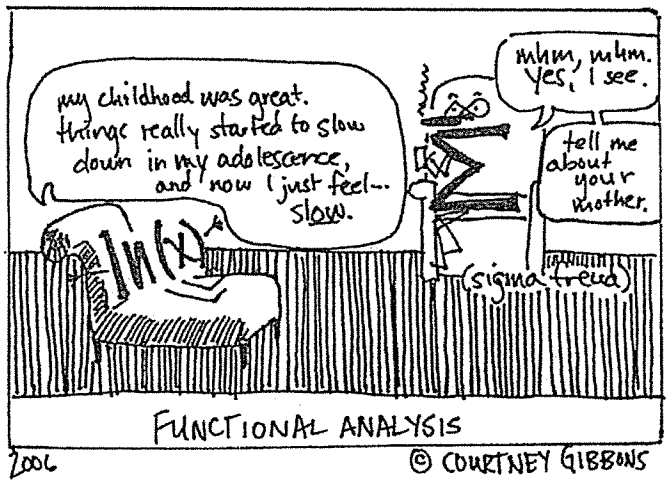
KEY

Math 126: Quiz 1

September 11, 2015

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Explain the difference between a *sequence* and a *series*

A sequence is an ordered list of terms

A series is a (usually infinite) ordered sum of a list of terms.

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2. True or false: If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=0}^{\infty} a_n$ converges.

False. Example: $\sum_{n=1}^{\infty} \frac{1}{n}$ Diverges, but $\lim_{n\to\infty} \frac{1}{n} = 0$

The converse is true.

3. Determine whether the following sequences converge, and if so, to what limit.

(a) $\left\{\frac{n-2}{n+1}\right\}_{n=1}^{\infty}$ $\quad -\frac{1}{2}, 0, \frac{1}{4}, \frac{2}{5}, \frac{3}{6}, \ldots \frac{100}{103} \ldots$

Both linear terms → compare coeff's. limit = 1

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(b) $\left\{\frac{n^2}{\sqrt{n^3+1}}\right\}_{n=1}^{\infty}$ $\quad$ Compare powers $\frac{n^2}{n^{1.5}}$ ← top is greater ⇒ Divergence

(c) $\left\{\frac{2^n}{n^{100}}\right\}_{n=1}^{\infty}$ $\quad$ $\frac{\text{Exponential}}{\text{Polynomial}}$ ← grows quicker ⇒ Divergence

(d) $\left\{\sqrt{n^2+n}-n\right\}_{n=1}^{\infty}$ $\quad a_n = \left(\sqrt{n^2+n} - n\right)\left(\frac{\sqrt{n^2+n}+n}{\sqrt{n^2+n}+n}\right)$

$$a_n = \frac{n^2+n-n^2}{\sqrt{n^2+n}+n} = \frac{n}{\sqrt{n^2+n}+n} \sim \frac{n}{2n}$$

limit $= \frac{1}{2}$

4. (a) Give *two* sigma-notation formulas for the series

$$\frac{1}{3}+\frac{1}{7}+\frac{1}{11}+\dots$$

with different starting indices.

$$\sum_{k=0}^{\infty} \frac{1}{4k+3} \quad \text{or} \quad \sum_{k=1}^{\infty} \frac{1}{4k-1}$$

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(b) Does the above series converge or diverge? Explain.

It Diverges, since the denominators do not space further apart. It is a harmonic type series.

$3\frac{1}{2}$

5. We showed in class that $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. What can we say about $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$?

$2\frac{1}{2}$

$$\frac{1}{1} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{2} + \frac{1}{\sqrt{5}} + \dots + \frac{1}{\sqrt{8}} + \frac{1}{3} + \dots + \frac{1}{4}, \dots$$

As $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ contains all terms of the harmonic series plus some others, it too must diverge.

6. Find the sum of each of the following series, or explain why no sum exists. It may help to write out the first few terms in each case.

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(a) $\sum_{n=0}^{\infty} \frac{2+3^n}{4^n}$ $= \sum_{n=0}^{\infty} \frac{2}{4^n} + \frac{3^n}{4^n}$ Each is Harmonic

$a = 2$ $a = 1$

$r = 1/4$ $r = 3/4$

$S = \frac{2}{1-1/4} + \frac{1}{1-3/4} = \frac{8}{3} + 4 = 20/3.$

(b) $\sum_{n=0}^{\infty} \frac{2^{2n}}{3^n}$ $= \sum_{n=0}^{\infty} \left(\frac{4^n}{3^n}\right)$ geometric, ratio is $\frac{4}{3}$

so <u>not</u> <u>convergent</u>

(c) $\sum_{n=0}^{\infty} \frac{n^2}{\sqrt[3]{n^6+2}}$ $\sum_{n=0}^{\infty} \frac{n^2}{\sqrt[3]{n^6+2}}$

$\lim_{n\to\infty} \frac{n^2}{\sqrt[3]{n^6+2}}$ compare effective powers $\frac{n^2}{n^2} \to 1$

Since $\lim_{n\to\infty} a_n \neq 0$ the series Diverges.

(d) $\sum_{n=0}^{\infty} \frac{(-4)^{n+2}}{5^n}$ $= \sum_{n=0}^{\infty} 16 \left(\frac{-4}{5}\right)^n$ $S = \frac{16}{1+4/5} = \frac{80}{9}.$

geometric! $a = 16$

$r = (-4/5)$