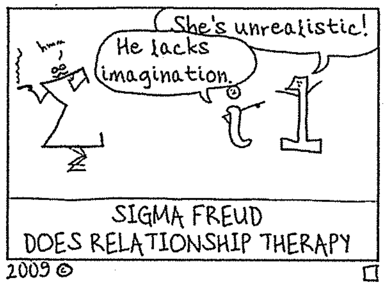
KEY

Math 126: Quiz 2

September ~~26, 2014~~ 25 2015

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Give an example of a *conditionally convergent* series, and explain why it is conditionally convergent.

Ex] $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ Converges by the Alt series test

But $\sum_{n=1}^{\infty} \frac{1}{n}$ Diverges

$\to$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is Conditionally Convergent.

2. Suppose that we have two series $\sum a_n$ and $\sum b_n$. We know that $\sum a_n$ is convergent, and we know that $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$. What can we conclude about $\sum b_n$?

Nothing. The LTC mandates that

$\lim_{n\to\infty} \frac{a_n}{b_n} = c$, $c \neq 0$. and finite

3. What is a p-series and when does one converge?

A p-series is one of the form $\sum_{n=1}^{\infty} \frac{1}{n^p}$ It converges when $p > 1$

4. For each of the following series, tell whether the series converges absolutely, converges contidionally, or diverges. In three of the cases, carry out an appropriate test in detail in order to justify your answer.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ $\frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}}$ $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0 \Rightarrow$ Convergent -

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ Diverges (p-series w $p = \frac{1}{2} < 1$) $\Rightarrow$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is Conditionally Convergent.

(b) $\sum_{n=1}^{\infty} \frac{2n}{\sqrt{n^5+2}} \sim \sum_{n=1}^{\infty} \frac{n}{\sqrt{n^5}} = \sum_{n=1}^{\infty} \frac{1}{n^{1.5}}$ convergent

$\lim_{n\to\infty} \frac{2n}{\sqrt{n^5+2}} \cdot \frac{n^{1.5}}{1} = \lim_{n\to\infty} \frac{2n^{2.5}}{\sqrt{n^5+2}} = 2 \neq 0$

$\Rightarrow$ Both series behave the same

$\to$ Both Converge

(c) $\sum_{n=4}^{\infty} \frac{3^n}{(n-2)!}$

RATIO TEST

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{3^{n+1}}{(n-1)!} \cdot \frac{(n-2)!}{3^n} \right|$

$= \lim_{n\to\infty} \frac{3}{n-1} = 0 < 1$

$\Rightarrow \sum_{n=4}^{\infty} \frac{3^n}{(n-2)!}$ Converges Absolutely

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{2^{n+1}}$ $= \frac{1}{2} \displaystyle\sum_{n=1}^{\infty} \frac{n^n}{2^n}$

Root TEST $\displaystyle\lim_{n\to\infty} \sqrt[n]{\left|\frac{n^n}{2^n}\right|}$

$= \displaystyle\lim_{n\to\infty} \frac{n}{2} \to \infty \to$ Divergence

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-4)^n}{n^4}$ $= \displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{4^n}{n^4}$ $\displaystyle\sum_{n=8}^{\infty} \frac{(-4)^n}{n^4}$

$b_n \to \infty$ as $n \to \infty$ $\Rightarrow$ Divergent

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 4 \cdot 7 \cdots (3n-2)}{(n+2)!}$

Ratio Test! $\displaystyle\lim_{n\to\infty} \left| \frac{1 \cdot 4 \cdot 7 \cdots (3n-2)(3n+1)}{(n+3)!} \cdot \frac{(n+2)!}{1 \cdot 4 \cdot 7 \cdots (3n-2)} \right|$

$= \displaystyle\lim_{n\to\infty} \left| \frac{3n+1}{n+3} \right| = 3 > 1$ $\Rightarrow$ Divergent

5. Carry out the ratio test for the following series and determine whether it converges:

$$\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$$

(Note: This one gets tricky. Feel free to use a calculator to give you some direction on the limit produced by the ratio test.)

Ratio test

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(n+1)^{n+1}}{(2n+2)!} \cdot \frac{(2n)!}{n^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)^{n+1}}{n^n \cdot (2n+2)(2n+1)} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)^{n+1}}{2(n^n)(n+1)(2n+1)} \right|$$

$$= \lim_{n\to\infty} \left| \underbrace{\left(\frac{n+1}{n}\right)^n}_{\to e} \cdot \underbrace{\frac{1}{2\cdot(2n+1)}}_{\to 0} \right| = 0$$

$\Rightarrow$ The series is convergent

6. Pick up a bonus questions sheet on your way out.