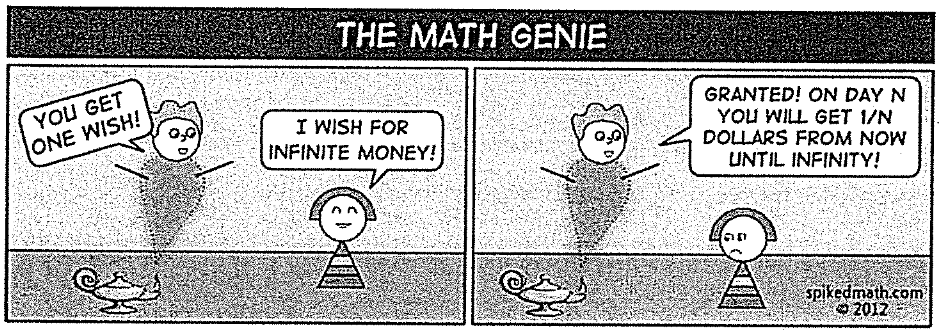
KEY

Math 126: Quiz the Third

October 12, 2015

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Find the values of x for which the series

$$\sum_{n=0}^{\infty} \frac{1}{n}\left(\frac{x+2}{3}\right)^n$$ Converges

Be sure to check your endpoints!!

Ratio test!

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$$\lim_{n\to\infty}\left|\frac{1}{n+1}\left(\frac{x+2}{3}\right)^{n+1}\cdot n\left(\frac{3}{x+2}\right)^n\right|$$

$$= \lim_{n\to\infty}\left|\frac{n}{n+1}\left(\frac{x+2}{3}\right)\right| = \left|\frac{x+2}{3}\right| < 1$$

$$\Rightarrow -1 \underset{?}{\le} \frac{x+2}{3} \underset{?}{\le} 1 \rightarrow -3 \underset{?}{\le} x+2 < 3$$

$$\rightarrow -5 < x < 1$$

If $x = -5$: $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ C

If $x = 1$: $\sum_{n=1}^{\infty} \frac{1}{n}$ Diverge

So $\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{x+2}{3}\right)^n$ converges for $x \in [-5, 1)$

2. (a) Find a series representation for

$$f(x) = \frac{4}{2+x}.$$

List at least the first five non-zero terms.

$$f(x) = \frac{4}{2+x} = \frac{4}{2(1+\frac{x}{2})} = \frac{2}{1+\frac{x}{2}}$$

$$= 2 - x + \frac{x^2}{2} - \frac{x^3}{4} + \frac{x^4}{8} - \cdots = 2\sum (-1)^n \left(\frac{x}{2}\right)^n$$

(b) What is the radius of convergence for your series?

$$\left|\frac{x}{2}\right| < 1 \Rightarrow \text{Radius} = \underline{2}$$

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(c) Use calculus to find a series for

$$g(x) = \frac{-4}{(2+x)^2}$$

$$g(x) = f'(x) = -1 + x - \frac{3x^2}{4} + \frac{4x^3}{8} - \frac{5x^4}{16} + \cdots$$

3. (a) Find a series representation for $f(x) = e^x$.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

(b) Find a series representation for $g(x) = xe^{3x}$

$$e^{3x} = 1 + 3x + \frac{3^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{3^4x^4}{4!} + \cdots$$

$$g(x) = xe^{3x} = x + 3x^2 + \frac{3^2x^3}{2!} + \frac{3^3x^4}{3!} + \frac{3^4x^5}{4!} + \cdots$$

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(c) Find $g^{(7)}(0)$, that is, the seventh derivative of g evaluated at $x = 0$.

focus on $\left(\frac{3^6x^7}{6!}\right)$

$0 \leftarrow \quad \rightarrow 0$

$$g^{(7)}(0) = \frac{3^6 \cdot 7!}{6!} = 7 \cdot 3^6$$

4. Find exact values for the following expressions:

(a) $1 - \frac{\pi^2}{2^2 * 2!} + \frac{\pi^4}{2^4 * 4!} - \frac{\pi^6}{2^6 * 6!} + \ldots$

$f(x) = \cos(x) \Big|_{x=\frac{\pi}{2}} \rightarrow \quad \cos\frac{\pi}{2} = 0$

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(b) $3 - \frac{3}{2} + \frac{3}{3} - \frac{3}{4} + \ldots$

$3\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\right)$

$= 3(\ln 2) = \quad 3\ln 2 = \ln 8$

5. Let $f(x)$ be an even function. Prove that $f'(x)$ is an odd function.

Pf 1)

If f is even then $f(-x) = f(x)$

Then differentiate both sides

$-f'(-x) = f'(x)$

Chain rule

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or $f'(-x) = -f'(x)$

Pf 2)

If f is even and $f(x) = \sum_{n=0}^{\infty} c_n x^n$, then $c_{2n+1} = 0$ for all n

all odd coeff's are 0

$f'(x) = \sum_{n=0}^{\infty} n c_n x^{n-1}$, then $c_{2n} = 0$

all even coeff's are 0

$\Rightarrow f'(x)$ is odd