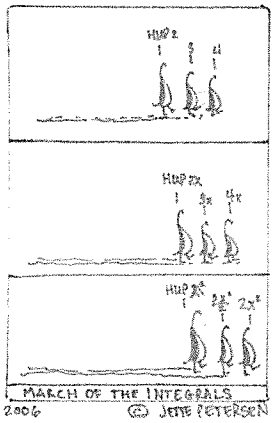
Math 126: Quiz 4
November 6, 2015

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Find FOUR of the following FIVE antiderivatives. Please clearly indicate which one is not to be graded. If you fail to indicate, I will choose one at random.

(a)

$$\int 4x\cos(x^2+1)\,dx$$

Let $u = x^2+1$

$du = 2x\,dx$

$$\hookrightarrow \int 2\cos u\,du$$

$$= 2\sin u + C$$

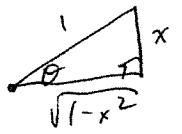
$$= 2\sin(x^2+1) + C$$

(b)

$$\int \frac{x^2\,dx}{\sqrt{1-x^2}}$$

Let $x = \sin\theta$

$dx = \cos\theta\, d\theta$



$$\int \frac{x^2}{\sqrt{1-x^2}}\,dx = \int \frac{\sin^2\theta\cos\theta}{\sqrt{1-\sin^2\theta}}\,d\theta = \int \sin^2\theta\, d\theta = \int \frac{1-\cos 2\theta}{2}\,d\theta$$

$$= \frac{\theta}{2} - \frac{\sin 2\theta}{4} + C$$

$$= \frac{\theta}{2} - \frac{\sin\theta\cos\theta}{2} + C$$

$$= \arcsin x - \frac{x\sqrt{1-x^2}}{2} + C$$

(c)

$$\int x^2 \sin(2x)\,dx$$

I.B.P. $\int x^2 \sin(2x)\,dx = -\frac{x^2\cos 2x}{2} + \int x\cos 2x\,dx$

$u = x^2$ $\quad dv = \sin 2x$ $\quad$ $u = x$ $\quad dv = \cos 2x\,dx$

$du = 2x\,dx$ $\quad v = -\frac{\cos 2x}{2}$ $\quad$ $du = dx$ $\quad v = \frac{1}{2}\sin 2x$

$$= -\frac{x^2\cos 2x}{2} + \frac{x}{2}\sin 2x - \frac{1}{2}\int \sin 2x\,dx$$

$$= -\frac{x^2}{2}\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos(2x) + C.$$

(d)

$$\int \frac{2x}{x^2+3x+2}\,dx$$

I. B. Partial fractions

$$\int \frac{2x}{x^2+3x+2}\,dx = \int \frac{2x}{(x+1)(x+2)}\,dx = \int \frac{A}{x+1} + \frac{B}{x+2}\,dx = \int \frac{-2}{x+1} + \frac{4}{x+2}\,dx$$

$$= -2\ln|x+1| + 4\ln|x+2| + C.$$

$A(x+2) + B(x+1) = 2x$

$A + B = 2$

$2A + B = 0$

$-A = 2,\ A = -2,$

$B = 4.$

(e)

$$\int \ln(2x+5)\,dx$$

1st Sub $u = 2x+5 \rightarrow \frac{1}{2}\int \ln u\,du$

$du = 2\,dx$ $= \frac{1}{2}(u\ln u - u)\,du$

$= \frac{1}{2}\left((2x+5)\ln(2x+5) - (2x+5)\right) + C$

Alt... DRY IBP

$$\int \ln(2x+5)\,dx = x\ln(2x+5) - \int \frac{2x}{2x+5}\,dx$$

$\frac{2x}{2x+5} = \frac{2x+5-5}{2x+5} = 1 - \frac{5}{2x+5}$

$u = \ln(2x+5) \quad dv = dx$

$du = \frac{2}{2x+5}\,dx \quad v = x$

$= x\ln(2x+5) - \int 1 - \frac{5}{2x+5}\,dx$

$= x\ln(2x+5) - x + \frac{5}{2}\ln(2x+5) + C$

(note our answers are equivalent up to the value of C)

2. Which of the following is $\int x^4\sqrt{1-x^2}\,dx$ (Do NOT try to compute the integral!). Explain your choice. ↳ eff power = x^5

   (a) $\frac{1}{48}(x\sqrt{1-x^2}(8x^2-2x-3)+3\arcsin(x))+C$

   (b) $\frac{1}{48}(x\sqrt{1-x^2}(8x^3-2x^2-3)+3\arcsin(x))+C$

   (c) $\frac{1}{48}(x\sqrt{1-x^2}(8x^4-2x^2-3)+3\arcsin(x))+C$

   ↳ eff. power = x^6.

3. (Bonus) Which topics of application (biology, chemistry, economics, mathematics, probability, or others) would you like to see covered in the remaining weeks of the semester?

4. (Bonus) Give a creative nomenclature for a 'group of integrals'.