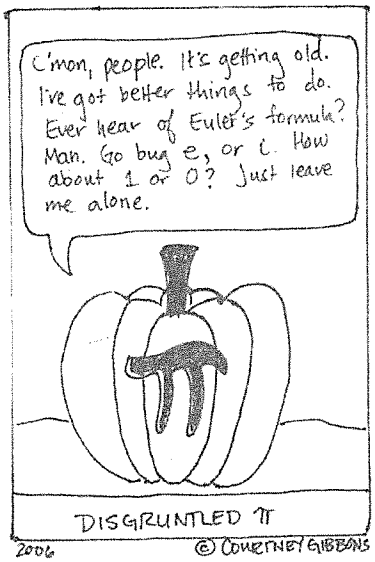
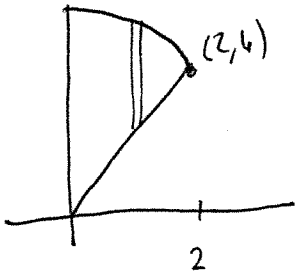
Math 126: Quiz 5
November 20, 2015

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Find the area in the first quadrant bound by $y = 3x$, $y = 10 - x^2$, and $x = 0$.



Top: $y = 10 - x^2$

Bottom: $y = 3x$

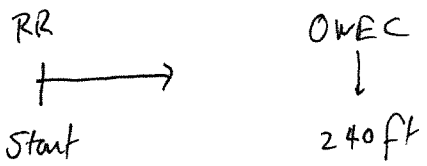
$$\text{Area} = \int_0^2 10 - x^2 - 3x \, dx$$

$$= 10x - \frac{x^3}{3} - \frac{3x^2}{2} \Big]_0^2$$

$$= 20 - \frac{8}{3} - 6 = 14 - \frac{8}{3} = \frac{34}{3}.$$

2. The Road Runner is running along at 80 ft/sec when he enters a school zone. The speed limit in the zone is 30 ft/sec. He sees Officer W. E. Coyote ~~400~~ 240 feet ahead of him, so he begins to decelerate at a rate of 10 ft/sec^2. If he is above the speed limit when he passes the officer, he will get a ticket. When will he pass the officer, and does he slow down in time?

(Find the time and speed when the Road Runner is at position ~~400~~ 240 feet, with an initial velocity of 80 ft/sec and a deceleration of 10 ft/sec^2).



~~$a(t) = 80$ ft/~~

$a(t) = -10$ ft/s^2

$v(t) = -10t + c$ ft/s

$v(0) = 80 \rightarrow v(t) = -10t + 80$ ft/s

$s(t) = -5t^2 + 80t + C$ $\quad s(0) = 0 \Rightarrow$

$s(t) = -5t^2 + 80t$

Find when $s = 240$

$240 = -5t^2 + 80t \rightarrow -5(t^2 - 16t + 48) = 0$

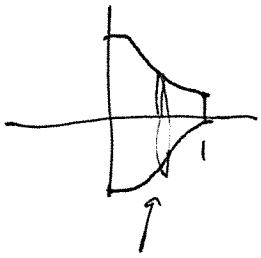
$\Rightarrow t = 4$ or $t = 12$

↑ use this, as it is the first pass

$v(4) = -10(4) + 80$

$\Rightarrow v(4) = 40$ too fast! Ticket!

3. Find the volume generated when the area bound by $y = \frac{1}{1+x^2}$, $x = 1$ and the coordinate axes is rotated about the x-axis.



Disks

$$\text{Volume} = \int_0^1 \pi \left(\frac{1}{1+x^2}\right)^2 dx$$

$x = \tan\theta \qquad 0 \le \theta \le \frac{\pi}{4}$

$dx = \sec^2\theta\, d\theta$

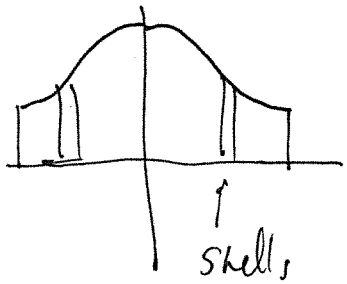
$$\int_0^{\pi/4} \pi \left[\frac{1}{(1+\tan^2\theta)^2}\right] \sec^2\theta\, d\theta$$

$$= \int_0^{\pi/4} \pi \left[\frac{1}{\sec^2\theta}\right] d\theta = \int_0^{\pi/4} \pi \cos^2\theta\, d\theta$$

$$= \pi \int_0^{\pi/4} \frac{1+\cos 2\theta}{2}\, d\theta = \pi \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\pi/4}$$

$$= \pi\left[\frac{\pi}{8} + \frac{1}{4}\right] = \frac{\pi^2}{8} + \frac{\pi}{4}.$$

4. Find the volume generated when the area bound by $y = \frac{1}{1+x^2}$, $x = 1$ and the coordinate axes is rotated about the y-axis.



Shells

$$\int_0^1 2\pi x \frac{1}{1+x^2}\, dx$$

let $u = 1+x^2$

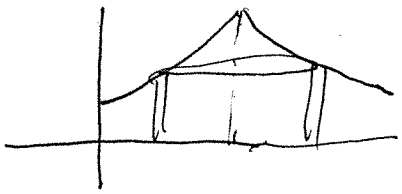
$du = 2x\, dx$

$$= \int_1^2 \pi \frac{1}{u}\, du$$

$$= \pi \ln|u| \Big]_1^2 = \pi \ln 2 - \pi \ln 1$$

($\ln 1 = 0$)

5. Find the volume generated when the area bound by $y = e^x$, $x = 1$ and the coordinate axes is rotated about the line $x = 1$.



Shells! ✓radius

$$\int_0^1 2\pi(1-x)e^x\,dx$$

IBP $u = 1-x$ $dv = e^x dx$

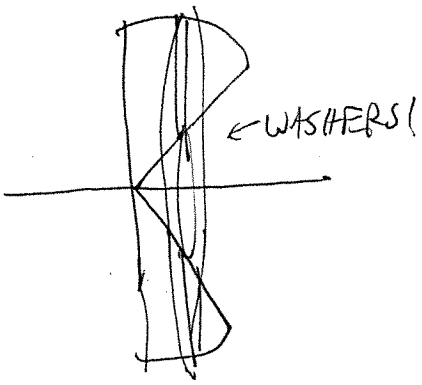
$du = -dx$ $v = e^x$

$$= 2\pi\left[(1-x)e^x - \int -e^x\,dx\right]$$

$$= 2\pi\left[(1-x)e^x + e^x\right]_0^1 = 2\pi\left[0 + e - (1+1)\right]$$

$$= 2\pi(e-2)$$

6. Set up, but don't evaluate, an integral for the volume generated when the area in the first quadrant bound by $y = 3x$, $y = 10 - x^2$, and $x = 0$ is rotated about the x-axis.



$$\int_0^2 \pi\left[(10-x^2)^2 - (3x)^2\right]dx$$