KEY

Math 126: Quiz 6

December 9, 2015

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!

1. Find the length of the curve $y = 3 + 2x^{\frac{3}{2}}$ from $(1, 5)$ to $(4, 19)$

$y' = \frac{2\cdot 3}{2} x^{1/2} = 3\sqrt{x}$

$\text{length} = \int_1^4 \sqrt{1 + 9x}\, dx$

$= \frac{2}{3} \cdot \frac{1}{9} (1 + 9x)^{3/2} \Big|_1^4$

$= \frac{2}{27} [37^{3/2} - 2^{3/2}]$

2. (a) Find the area of the surface generated by rotating the portion of the curve $y = x^3$ between $x = 0$ and $x = 3$ about the x-axis.

$S.A. = \int 2\pi r\, ds$

$= \int_0^3 2\pi x^3 \sqrt{1 + (3x^2)^2}\, dx = \int_0^3 2\pi x^3 \sqrt{1 + 9x^4}\, dx$

$u = 1 + 9x^4$ ll $0 + 9(0)^4 = 1$

$du = 36x^3 dx$ ul $= 1 + 9(3)^4 = 730$

$\rightarrow SA = \frac{2\pi}{36} \int_1^{730} \sqrt{u}\, du = \frac{2\pi}{36} \cdot \frac{2}{3} u^{3/2} \Big|_1^{730} = \frac{\pi}{27}(730^{3/2} - 1^{3/2})$

(b) Set up, but don't evaluate, the integral for the surface area generated by rotating that same portion of the curve about the y-axis.

$\int_0^3 2\pi x \sqrt{1 + 9x^4}\, dx$

or $\int_0^3 2\pi \sqrt[3]{y} \sqrt{1 + \left(\frac{1}{9} y^{-4/3}\right)}\, dy$

3. Find the value, if it exists, of

$$\int_0^\infty xe^{-x^2}\,dx$$

$$\lim_{b\to\infty}\int_0^b xe^{-x^2}dx = -\frac{1}{2}\lim_{b\to\infty}\int_0^b e^u\,du$$

$u = -x^2$

$du = -2x\,dx$

$$= \frac{-1}{2}e^{-x^2}\Big]_0^b$$

$$= 0 + \frac{1}{2} = \frac{1}{2}$$

4. Without calculating, would you expect

$$\int_2^\infty \frac{x^2}{\sqrt{1+x^5}} \sim \int \frac{x^2}{x^{5/2}} \to \int \frac{1}{x^{1/2}}$$

to converge or diverge, and why?

Diverge. It behaves like a p-integral with $p = 1/2 < 1$.

5. Suppose that we have society that elects 1200 new members each year. In addition, the society loses 15% of its members each year.

(a) Give a differential equation for the membership M as it changes relative to time t.

$\frac{dM}{dt} = 1200 - .15M$

$\frac{dM}{dt} = .15(M - 8000)$ → $M = 8000 + Be^{-.15t}$

(b) If there are 3000 members to begin with, how many will there be after 3 years?

$M(0) = 3000 \Rightarrow B = 3000 - 8000 = -5000$

$M = 8000 - 5000e^{-.15t}$

$M(3) = 8000 - 5000e^{-.45} = 4811$

(c) If there are 3000 members to begin with, how many will be in the society long-term?

long term → 8000 as the constant Dropsoff

(d) If there are 10,000 members to begin with, how many will there be in the society long-term?

10000 start

$B(t) = 8000 + 2000e^{-.15t}$

→ B → 8000 still

6. A pair of species exist on an island and are competing for the same limited (but renewable) resource for survival. The differential equations governing the system are as follows.

$$\frac{dA}{dt} = r_1 A + s_1 A^2 + t_1 AB$$
$$\frac{dB}{dt} = r_2 B + s_2 B^2 + t_2 AB$$

Are the constants $(r_1, s_1, t_1, r_2, s_2, t_2)$ positive or negative? Explain your reasoning biologically.

$r_1, r_2 > 0$ → These follow a logistic pattern.
$s_1, s_2 < 0$ → The population can get larger, but not too large

$t_1, t_2 < 0$ ⇒ When both species are present, it adversely affects both populations!

7. Which concepts are you most comfortable about heading into the final? Where do you need the most work?