These questions below are culled from old quizzes and exams and represent the level of difficulty that you'll face on Friday's exam. Note that the exam will have both *conceptual* as well as *computational* questions, so be sure to be able to answer the types of questions 1-5. (Note that we haven't done this formally on paper, but we ask many many of these questions in class every day). Note that this list is not exhaustive, and that the questions on the exam will necessarily be different than the ones here. These should serve as a good indicator of how well you comprehend the material from the first few weeks of class.

1. Explain the difference between a *sequence* and a *series*

2. Give two examples of *geometric* series, one which converges and one which does not.

3. Suppose that $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ are series such that $\lim_{n\to\infty} \frac{a_n}{b_n} = c$, with c finite and $c \neq 0$. What conclusions can we draw about the two series.

4. Suppose that we have a series $\sum_{n=0}^{\infty} a_n$ for which $a_n \leq \frac{1}{n}$ for all n. What conclusions can we draw about $\sum_{n=0}^{\infty} a_n$?

5. Suppose that we have a series $\sum_{n=0}^{\infty} a_n$ for which $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$. What conclusions can we draw based on the value of L?

6. Determine whether the following sequences converge (and the limits, if they exist). Give a brief justification for your answers. (Write out the first few terms of each, if you find it helpful).

   (a) $\left\{ \frac{(-1)^{n+1}}{\sqrt{n}} \right\}_{n=1}^{\infty}$

   (b) $\left\{ \frac{2n^2}{\sqrt{n^4+1}} \right\}_{n=1}^{\infty}$

   (c) $\left\{ \frac{n!}{(n-1)!} \right\}_{n=3}^{\infty}$

(d) $\left\{\frac{\ln(n)}{n}\right\}_{n=1}^{\infty}$

7. A ball bounces from a height of 5 feet. On each bounce, it returns to 3/4 of its height. Show that the total distance travelled by the ball is finite, and find the value.

8. (a) Write the following series in sigma notation.

$$\frac{1}{2}+\frac{1}{7}+\frac{1}{12}+\frac{1}{17}+\dots$$

   (b) Does the above series converge? Why or why not?

   (c) Write the following series in sigma notation, and determine whether or not it converges.

$$\frac{1}{2}-\frac{1}{7}+\frac{1}{12}-\frac{1}{17}+\dots$$

9. Determine the convergence (and sum) or divergence of each of the following series. Provide a justification for your answers. (Again, writing out the first few terms may help).

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{3^n+4^n}{5^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n+3}$

   (c) $\displaystyle\sum_{n=1}^{\infty}\frac{2n^2}{\sqrt{n^4+1}}$

   (d) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{2n}}$

10. For each of the six series presented, tell whether the series converges absolutely, converges conditionally, or diverges. In *three* of the cases, carry out an appropriate test to justify your response.

   (a) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n\,(2n+1)}{n\sqrt{n}}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{\ln(n+1)}{2^n}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-8)^n}{3^n n!}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(n+2)!}{2^{n^2}}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n+4}{4^n+3}$

(f) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{\sqrt{2n^8+5}}{n^4+n^2}\right)^n$

11. For which values of x does the series

$$\sum_{n=0}^{\infty} \frac{n+1}{n^2}(x+2)^n$$

converge? Be sure to check endpoints!

12. Find a series representation for

$$f(x) = \frac{2x}{3-2x}.$$

For which values of x does your series converge?