Examples

1) Difference b/w sequence + series?

$\boxed{\text{Series}}$ $\{a_n\}_{n=1}^{\infty}$

- Series is a sum of terms in a sequence
- Terms need to get smaller and approach $\emptyset$ in order to converge

$\boxed{\text{Sequence}}$ $\sum_{n=1}^{\infty} a_n$

- Sequence is an ordered list of terms
- Need to approach some number in order to converge

3) $\lim \frac{a_n}{b_n} = c \neq 0$, use the Limit Comparison Test

$\sum_{n=1}^{\infty} a_n$ vs $\sum_{n=1}^{\infty} b_n$ = $\lim \frac{a_n}{b_n} = c \neq 0$ then behave the same.

5) $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$ where
$L<1$ Converge
$L>1$ Diverge
$L=1$ can't tell

Examples (2)

7) *practical application of series.

<u>Purpose</u>:
Show that distance is finite and find total distance

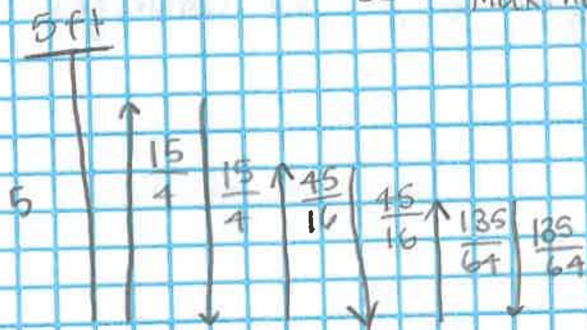
$$5 - \frac{15}{4} + \frac{15}{4} - \frac{45}{16} + \frac{45}{16} - \frac{135}{64}$$

- This is a geometric series ; going to get double heights

$$5 + 2\left(\frac{15}{4} + \frac{45}{16} + \frac{135}{64} + \cdots\right)$$

$$\hookrightarrow 5 + 2\left(\frac{\frac{15}{4}}{1-\frac{3}{4}}\right) = 5 + 2(15) = 35\text{ ft} \qquad \frac{a_n}{1-r}$$

35 = Max height + 2(each distance/term)



* Draw Picture to understand what is going on

9) a) $\frac{3^n + 4^n}{5^n}$ → looks geo. but seperate 2 terms

$$\sum_{n=1}^{\infty} \frac{3^n}{5^n} + \sum_{n=1}^{\infty} \frac{4^n}{5^n} \Rightarrow \frac{\frac{3}{5}}{1-\frac{3}{5}} + \frac{\frac{4}{5}}{1-\frac{4}{5}} = \boxed{5.5}$$

$C = 5.5 \neq 0$ so behave like $\frac{3^n}{5^n}$ which Diverges

b) $\sum \frac{1}{n+3}$ vs $\sum \frac{1}{n}$

$$\lim_{n\to\infty} \frac{\frac{1}{n+3}}{\frac{1}{n}} \to \frac{n}{n+3} = 1 \text{, both Diverge}$$

c) $\sum_{n=1}^{\infty} \frac{2n^2}{\sqrt{n^4+1}}$ remember previous examples:

$\lim_{n\to\infty} = 2 \neq 0$, therefore series diverges.

d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}}$ → converge b/c alternator and large denominator

$$1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \cdots$$

now we can see its an alternating, geometric series

$$S = \frac{1}{1-(-\frac{1}{4})} = \frac{4}{5} < 1$$ so converges.

Examples (3)

1) $\frac{n+1}{n^2}(x+2)^n$ use ratio test and you'll get

$$\lim_{n\to\infty} |x+2| < 1$$

$$-1 < x+2 < 1 \longrightarrow \boxed{\begin{matrix} -3 < x < -1 \\ ? \quad ? \end{matrix}}$$

if $x = -3$ $\sum \frac{n+1}{n^2}(-1)^n = C$

if $x = -1$ $\sum \frac{n+1}{n^2}(1)^n = D$

$\Big\} -3 \leq x < 1$

2)

$$f(x) \frac{\frac{2x}{3}}{1-\frac{2x}{3}} = \frac{2x}{3} + \frac{4x^2}{9} + \frac{8x^3}{27} + \cdots$$

$$\boxed{\left|\frac{2x}{3}\right| < 1}$$

2 GEOMETRIC SERIES: $A + AR + AR^2 + AR^3 + \cdots \leftarrow R =$ RATIO OF TERMS

EX C: $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \cdots$

D: $1 + \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \cdots$

4 $\sum_{n=1}^{\infty} A_n \le \sum_{n=1}^{\infty} \frac{1}{n}$ NO CONCLUSION CAN BE DRAWN

D

6 (A) $\left\{\frac{(-1)^{n+1}}{\sqrt{n}}\right\}_{n=1}^{\infty}$ $1, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{4}},$

$L=0$

(B) $\left\{\frac{2n^2}{\sqrt{n^4+1}}\right\}_{n=1}^{\infty} \sim \frac{n^2}{n^2} \rightarrow$ COMPARE COEFFICENTS $\rightarrow L=2$

(C) $\left\{\frac{n!}{(n-1)!}\right\}_{n=3}^{\infty}$ $3, 4, 5, 6, \ldots$ L = DNE

(D) $\left\{\frac{\ln(n)}{n}\right\}_{n=3}^{\infty} \leftarrow \ln(n)$ GROWS SLOWLIER THAN $n \rightarrow L=0$

8 (A) $\frac{1}{2} + \frac{1}{7} + \frac{1}{12} + \frac{1}{17} + \cdots$

$\sum_{n=1}^{\infty} \frac{1}{5n-3} \sim \sum \frac{1}{n}$

(B) DIVERGES IN COMPARISON TO $\sum \frac{1}{n}$

(C) $\frac{1}{2} - \frac{1}{7} + \frac{1}{12} - \frac{1}{17} + \cdots$

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{5n-3} \rightarrow$ CONVERGES BY THE ALTERNATING SERIES TEST

10 (A) $\sum_{n=2}^{\infty} \frac{(-1)^n (2n+1)}{n\sqrt{n}}$

ALTERNATING TERMS $\rightarrow 0 \rightarrow \underline{C}$ C

$\sum_{n=2}^{\infty} \frac{2n+1}{n\sqrt{n}} \sim \sum \frac{1}{\sqrt{n}}$ CONDITIONALLY CONVERGENT BECAUSE

D $\leftarrow$ D IT ONLY CONVERGES BECAUSE ALTERNATING

(B) $\sum_{n=0}^{\infty} \frac{\ln(n+1)}{2^n} < \sum_{n=0}^{\infty} \frac{n+1}{2^n}$ RATIO TEST

$\lim_{n\to\infty} \left|\frac{A_{n+1}}{A_n}\right| = \lim_{n\to\infty} \left|\left(\frac{n+2}{2^{n+1}}\right)\left(\frac{2^n}{n+1}\right)\right| = \lim_{n\to\infty} \left|\left(\frac{1}{2}\right)\left(\frac{n+2}{n+1}\right)\right| = \frac{1}{2} \rightarrow$ ABSOLUTELY CONVERGENT

(C) $\sum_{n=0}^{\infty} \frac{(-8)^n}{3^n n!}$ RATIO TEST

$\lim_{n\to\infty} \left|\left(\frac{(-8)^{n+1}}{3^{n+1}(n+1)!}\right)\left(\frac{3^n n!}{(-8)^n}\right)\right| = \lim_{n\to\infty} \frac{-8}{3(n+1)} = 0 \rightarrow$ CONVERGES

(D) $\sum_{n=0}^{\infty} \frac{(n+2)!}{2^{n^2}}$ RATIO TEST

$\lim_{n\to\infty} \left|\left(\frac{(n+3)!}{2^{n^2+2n+1}}\right)\left(\frac{2^{n^2}}{(n+2)!}\right)\right| = \lim_{n\to\infty} \left|\frac{n+3}{2^{2n+1}}\right| = 0 \rightarrow$ ABSOLUTELY CONVERGENT

(E) COMPARE TO $3^n/4^n \rightarrow$ CONVERGES

(F) ROOT TEST $L = \sqrt{2}$, DIVERGES

12 $F(x) = \frac{2x}{3-2x} = \frac{2x/3}{1-2x/3}$

$= \frac{2x}{3} + \frac{4x^2}{9} + \frac{8x^3}{27} + \cdots$

CONVERGES FOR $\left|\frac{2x}{3}\right| < 1$