

Math 126-Week 2 Homework

Due September 12 in class

Provide thorough written responses to the exercises below. Use complete sentences for your responses, and explain all of your steps in each problem. Make your work neat and presentable (rewrite your solutions from scratch paper if need be). Note: You are free to work with each other, but do not consult any other resources besides your book, your notes, and your professor.

1. Let

$$a_n = \sqrt{n^2 + 6n} - \sqrt{n^2 + 2n}.$$

Determine if a_n converges, and to which value.

2. Find a formula for a_n for the sequence

$$\left\{ 1, \frac{4}{7}, \frac{9}{17}, \frac{16}{31}, \frac{25}{49}, \frac{36}{71}, \dots \right\}.$$

Does this sequence converge? If so, to what?

3. Determine whether

$$\sum_{n=5}^{\infty} \frac{1}{\sqrt{n-2}} - \frac{1}{\sqrt{n}}$$

converges, and if so, to what value.

4. Let

$$a_n = \left(1 + \frac{1}{n} \right)^n.$$

Determine numerically (i.e., use your calculator) whether a_n converges, and to which value. Show your result algebraically as well, by considering the sequence $b_n = \ln(a_n)$.

5. (a) Alice and Bob are going to play the following game: Each will take turns tossing a fair coin. The first one to toss a 'Heads' wins. Alice plays first. What is the probability that she wins?
- (b) Same players, but now, each will toss again if they toss a 'Heads'. First to toss two 'Heads' in a row wins.
- (c) (Bonus) Same as before, but they will once again take turns. First to toss two 'Heads' wins.