Determine Convergence or Divergence

1a) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n + n^2}{2^{2n} + n^3} \leftarrow a_n$

We compare this series with $\displaystyle\sum \frac{3^n}{2^{2n}} \leftarrow b_n$ using the LCT.

$$\lim_{n\to\infty} \left|\frac{a_n}{b_n}\right| = \lim_{n\to\infty} \left|\frac{3^n + n^2}{2^{2n}+n^3} \cdot \frac{2^{2n}}{3^n}\right|$$

$$= \lim_{n\to\infty} \left|\left(\frac{3^n+n^2}{3^n}\right)\left(\frac{2^{2n}}{2^{2n}+n^3}\right)\right|$$ Both parts dominated by exponentials

$$= 1 \cdot 1 = 1 \neq 0.$$ so the series behave the same.

Since $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{2^{2n}} = \sum_{n=0}^{\infty} \frac{3^n}{4^n}$ Converges (geometric, $r = 3/4 < 1$)

$\displaystyle\sum_{n=0}^{\infty} \frac{3^n + n^2}{2^{2n}+n^3}$ also converges

b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$ Since $\ln(n) < n$, we have that

$$\frac{1}{\ln(n)} > \frac{1}{n}$$

So $\displaystyle\sum \frac{1}{\ln(n)} > \sum \frac{1}{n}$. By the comparison test, since

$\displaystyle\sum \frac{1}{n}$ Diverges,

$\displaystyle\sum \frac{1}{\ln(n)}$ also Diverges.

c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[3]{n} - 1}{n\sqrt{n+1}}$ With polynomials on top & bottom, this should behave like a p-series, with power $\dfrac{n^{1/3}}{n^{3/2}} = \dfrac{1}{n^{7/6}}$

LCT : $$\lim_{n\to\infty} \left|\frac{\sqrt[3]{n}-1}{n\sqrt{n+1}} \cdot \frac{n^{7/6}}{1}\right| = \lim_{n\to\infty}\left|\frac{n^{3/2} - n^{7/6}}{n\sqrt{n+1}}\right| = 1$$

So both series behave the same.

Since $\displaystyle\sum \frac{1}{n^{7/6}}$ converges $(p>1)$, $\displaystyle\sum \frac{\sqrt[3]{n}-1}{n\sqrt{n+1}}$ Converges.

d) $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2n-1)}{2 \cdot 5 \cdot 8 \cdot 11 \cdots (3n-1)}$

We use the Ratio Test here

$$\lim_{n\to\infty} \left| \frac{\cancel{1 \cdot 3 \cdot 5 \cdots (2n-1)}(2n+1)}{\cancel{2 \cdot 5 \cdot 8 \cdots (3n-1)}(3n+2)} \cdot \frac{\cancel{2 \cdot 5 \cdot 8 \cdots (3n-1)}}{\cancel{1 \cdot 3 \cdot 5 \cdots (2n-1)}} \right|$$

$= \lim_{n\to\infty} \left| \frac{2n+1}{3n+2} \right| = 2/3 < 1$, so our series is Abs. Convergent.

2. Let x be the sum of a geometric series beginning at 1.

$x = 1 + r + r^2 + r^3 + \cdots = \frac{1}{1-r}$ when $|r| < 1$

thus $x = \frac{1}{1-r} \rightarrow r = 1 - \frac{1}{x}$

$-1 < 1 - \frac{1}{x} < 1$

$-2 < -\frac{1}{x} < 0$

$\rightarrow 2 > \left(\frac{1}{x}\right) > 0$

$\rightarrow \underline{\frac{1}{2} < x < \infty}$ (or x is any real # greater than $\frac{1}{2}$.