This is an amended version of last year's exam (it reflects what we have done in class so far). I have also attached some more practice problems.

-Prof Balof

Math 126: Calculus 2 Exam the First

You have the class period to complete this closed-book-closed-notes-closed-colleague exam. Please read the statement below and sign it when you are finished.

I have not used my calculator on this examination except for arithmetic, trigonometric, logarithmic, and exponential functions. I certify that the work on this exam is my own and that I have not discussed the contents of this exam with anyone prior to my taking it.

Signature:

1. (10 points) What is the formula for $\int_a^b f(x)dx$ in terms of Riemann Sums? Be as explicit as possible in stating the formula.

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=0}^{n-1} f(a+\frac{b-a}{n}i)\frac{b-a}{n}$$

or

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(a+\frac{b-a}{n}i)\frac{b-a}{n}$$

2. (16 points) Find the area under the line $y = 2x + 3$ between $x = 1$ and $x = 4$ using

   (a) Geometry.

   (b) Calculus.

   Geometry: The figure is a trapezoid, with height 3 and bases 5 and 11. Its area is $3(5+11)/2 = 24$

   Calculus: $\int_1^4 2x + 3\ dx = x^2 + 3x|_1^4 = 16 + 12 - (1 + 3) = 24$

3. Solve the following. (8 points each)

(a) $\int \frac{x^3+1}{x^2} dx$

Break this up into $\int \frac{x^3}{x^2} + \frac{1}{x^2}\ dx = \int x + \frac{1}{x}\ dx = \frac{x^2}{2} + \ln|x| + C$.

(b) $\int (x+3)\cos(x^2+6x+5) dx$

Use u-substitution. $u = x^2{+}6x{+}5$, $du = 2x{+}6\ dx$. The integral becomes $\frac{1}{2}\int \cos(u)\ du = \frac{1}{2}\sin(u) + C = \frac{1}{2}\sin(x^2+6x+5) + C$.

(c) $\int_0^a \frac{xdx}{\sqrt{1-x^2}}$. What are the restrictions on a?

Use $u = 1 - x^2$, with $du = -2x\ dx$, transforming the integral to

$$\frac{-1}{2}\int u^{\frac{-1}{2}}\ du = \frac{-1}{2}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = -\sqrt{1-x^2} + C$$

In this problem, we cannot have $|a| > 1$, else the function would not be defined.

4. Consider the area under the curve $y = e^x$ between $x = 0$ and $x = \ln 3$

   (a) (10 points) Find c such that the line $x = c$ bisects this area. Is c greater than, less than, or equal to $\frac{1}{2}\ln 3$? Explain graphically.

   We compute the whole area

$$\int_0^{\ln 3} e^x \, dx = e^x|_0^{\ln 3} = 3 - 1 = 2$$

   and set up an integral to bisect it.

$$\int_0^{c} e^x \, dx = 1 \Rightarrow e^c - 1 = 1, e^c = 2, c = \ln 2$$

   This value is more than half of $\ln 3$ since the graph is increasing, more area will be bound by the second half of the interval.

   (b) (8 points each) Compute the volume generated by revolving this area about

      i. the x axis. (solve this integral...)

      ii. the y axis. (...but not this one.)

x-axis:

$$\int_0^{\ln 3} \pi (e^x)^2 \, dx = \pi \int_0^{\ln 3} e^{2x} \, dx = \frac{\pi}{2}(e^{2\ln 3} - e^0) = 4\pi$$

y-axis:

$$\int_0^{\ln 3} 2\pi x e^x \, dx$$

5. Find the ratio of area above the x axis to area below the x axis of the curve $y = x^3 - 9x$

   We could solve two integrals by finding intercepts, or could observe that the function is odd, so the ration would be 1/1

6. If we know that $\int_{-a}^{a} x^3 f(x)\, dx = 0$ for all values of a, what kind of function is $f(x)$? Explain.

   Since $x^3 f(x)$ is an odd function, $f(x)$ must be even.