Fall 2004
Math 126: Exam the Second

You have the class period to complete this closed-book-closed-notes-closed-colleague exam. Please read the statement below and sign it when you are finished.

I have not used my calculator on this examination except for arithmetic, trigonometric, logarithmic, and exponential functions. I certify that the work on this exam is my own and that I have not discussed the contents of this exam with anyone prior to my taking it.

Signature:

1. Find the volume of the solid generated when the region bounded by the curve $y = x \ln x$ between $x = 2$ and $x = 5$ is rotated about the y axis. (Hint: You'll want to use shells rather than solving for y. I'll 'sell' you the formula for shells if you need it.)

2. For each of the following (a-f), give specific strategies for integration. Then solve any **three** of the integrals. Please clearly indicate which three you've intended to solve.

(a)

$$\int \frac{2x}{x^2 - 4x + 3}\,dx$$

(b)

$$\int \sec^3(x)\tan^3(x)\,dx$$

(c)

$$\int \frac{x-1}{\sqrt{1-x^2}}\,dx$$

(d)

$$\int \frac{\sqrt{x}-1}{\sqrt{x}+1}\,dx$$

(e)

$$\int \frac{\ln x}{\sqrt{x}}\ dx$$

(f)

$$\int e^{2x} \cos(2x)\ dx$$

3. Find

$$\int_{\sqrt{3}}^{\infty} \frac{2}{1+x^2}$$

if it exists. Use this result to determine whether

$$\int_{\sqrt{3}}^{\infty} \frac{e^{-x}}{1+x^2}$$

converges or diverges.

4. Use the arc length formula to prove that the circumference of a circle of radius 1 is 2π.

More Practice Problems

1. Let $f(x) = \sqrt{x}e^x$. Determine the volume of the solid generated by revolving the area under $f(x)$ between $x = 1$ and $x = 5$ about the x axis.

2. Prove the formula for the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, Area=$\pi ab$.

3. Determine
$$\int \cos^3(x)\, dx$$

4. Determine
$$\int \frac{2x+1}{x^3+x^2}\, dx$$

5. Determine
$$\int x \arctan x\, dx$$

6. Give strategies for each of the following integrals. In each case, state the methods and any substitutions you would use.

   (a)
$$\int \frac{x^2+x}{x+2}\, dx$$

   (b)
$$\int \frac{x^2}{(4-x^2)^{3/2}}\, dx$$

   (c)
$$\int \frac{\sec^6(2x)}{\tan^2(2x)}\, dx$$

7. Determine
$$\int_2^\infty \frac{1}{x^2-1}\, dx$$

8. Use the comparison test to determine whether
$$\int_3^\infty \frac{1}{x\cos x}\, dx$$
converges or diverges.

9. Set up, but do not evaluate, the integrals necessary to determine the arc length and the surface area of the curve and solid in problem 1, and of the curve in problem 2.