Math 126: Exam the Third

You have the class period to complete this closed-book-closed-notes-closed-colleague exam. **READ ALL QUESTIONS AND DIRECTIONS CAREFULLY**. Please read the statement below and sign it when you are finished.

I have not used my calculator on this examination except for arithmetic, trigonometric, logarithmic, and exponential functions. I certify that the work on this exam is my own and that I have not discussed the contents of this exam with anyone prior to my taking it.

Signature:

1. Suppose that a sequence is defined by the recurrence relation $a_n = \frac{n}{n+1}a_{n-1}$, and $a_1 = 1$. Write out the first six terms. Does the sequence converge? If so, find the limit. Does the series $\sum_{n=1}^{\infty} a_n$ converge?

2. Suppose that a sequence a_n converges to a limit L. What conditions on L will ensure that $b_n = (a_n)^n$ also converges? BONUS: To what can b_n converge?

3. Find the sum of the series
$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n$$

4. For each of the six series below, determine whether the series is absolutely convergent, conditionally convergent, or divergent. For THREE of them, please justify your answer by using the appropriate test and giving all details. If using a comparison test, you may assume the convergence or divergence of a p-series.

   (a)
   $$\sum_{n=3}^{\infty} \frac{\ln(\ln n)}{\ln n}$$

   (b)
   $$\sum_{n=1}^{\infty} \frac{n!}{n^2}$$

   (c)
   $$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}2n}{3n+1}$$

   (d)
   $$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n+2}{2n^2+1}$$

(e)

$$\sum_{n=1}^{\infty} \frac{2^n}{n^2 + 3^n}$$

(f)

$$\sum_{n=1}^{\infty} (\arctan n)^{-n}$$

5. For which values x will the series

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n} x^n$$

converge? Be careful to check endpoints!

Do one of numbers 6 and 7

6. Find a series for $f(x) = \ln(1 - x^2)$. For what values is your series valid? Use this to explain the sign of each term in your series.

7. Use Maclaurin's formula to find the first three nonzero terms of the expansion of $f(x) = \frac{1}{\sqrt{1-x}}$. Use this to find series for $\frac{1}{\sqrt{1-x^2}}$ and $\arcsin(x)$ as well.