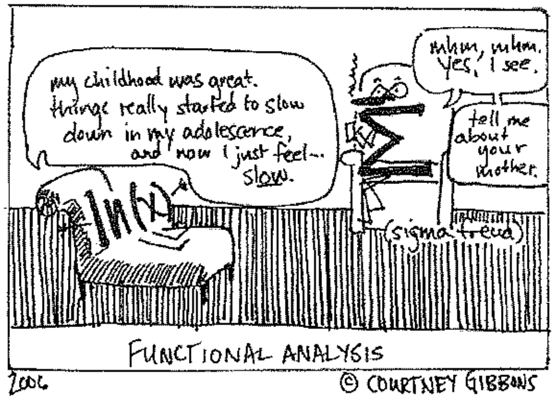
KEY

Math 126: Quiz 1

January 28, 2010

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Explain the difference between a *sequence* and a *series*

   A Sequence is a list of terms

   A series is an infinite sum of terms.

2. True or False. Give a brief justification in each case.

   (a) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

   True. The sequence of terms must converge to 0.

   (b) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

   False. $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, but $\frac{1}{n} \to 0$

   (c) All geometric series converge.

   False. We need the ratio to be less than 1.

3. Determine the convergence (and limit) or divergence of each of the following sequences. Give a brief justification for your answers. (Write out the first few terms of each, if you find it helpful).

(a) $\left\{\frac{2n}{3n+1}\right\}_{n=1}^{\infty}$ Compare coefficients in numerator, denominator

$\left\{\frac{2}{4}, \frac{4}{7}, \frac{6}{10}, \frac{8}{13} \ldots\right\}$ C $\rightarrow \frac{2}{3}$

(b) $\left\{\frac{(n+1)!}{n!}\right\}_{n=1}^{\infty}$

$\frac{(n+1)!}{n!} = \frac{(n+1)\,n!}{n!} = n+1$ Divergent

$\{2, 3, 4, 5, 6 \ldots\}$

(c) $\left\{\frac{n^2}{\sqrt{n^4+n}}\right\}_{n=1}^{\infty}$ power on top $\rightarrow 2$
power on bottom $\rightarrow 2$ $\rightarrow$ Convergent
$\rightarrow 1$

$\left\{\frac{1}{\sqrt{2}}, \frac{4}{\sqrt{18}}, \frac{9}{\sqrt{84}}, \frac{16}{\sqrt{260}}, \ldots\right\}$

(d) $\left\{\frac{(-1)^n n}{n+1}\right\}_{n=1}^{\infty}$

$\left\{-\frac{1}{2}, \frac{2}{3}, -\frac{3}{4}, \frac{4}{5}, -\frac{5}{6}, \ldots\right\}$ the terms are approaching both +1 and -1 (alternating)

$\rightarrow$ Divergent

4. Determine the convergence (and sum) or divergence of each of the following series. Provide a justification for your answers. (Again, writing out the first few terms may help).

   (a) $\sum_{n=1}^{\infty}(\sqrt{2})^n$ $= \sqrt{2}+2+2\sqrt{2}+4+4\sqrt{2}+8+\cdots$

   Geometric w/ratio $\sqrt{2}>1 \Rightarrow$ Divergent

   (b) $\sum_{n=1}^{\infty}\frac{1}{n+3}$

   $\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\cdots$

   Harmonic, just missing the first few terms

   $\Rightarrow$ Divergent

   (c) $\sum_{n=1}^{\infty}\frac{(-1)^n}{2^{2n}}$

   Geometric, ratio $= \frac{-1}{4}$

   $-\frac{1}{4}+\frac{1}{16}-\frac{1}{64}+\cdots$

   $\text{Sum} = \frac{a}{1-r} = \frac{-\frac{1}{4}}{1+\frac{1}{4}} = \frac{-1}{5}$

5. (Bonus) Alice and Bob are going to take turns tossing a fair coin, Alice goes first. Whoever gets the first head get $1 from the other. Who has the advantage? What is the probability that that person will win?

Alice has the advantage. Her probability of winning on the first toss is $\frac{1}{2}$, so her overall probability is greater

$$P(A\text{ wins}) = \underbrace{\tfrac{1}{2}}_{\text{A Heads}} + \underbrace{\tfrac{1}{2}}_{\text{A Tails}}\cdot\underbrace{\tfrac{1}{2}}_{\text{B Tails}}\cdot\underbrace{\tfrac{1}{2}}_{\text{A Heads}} + \underbrace{\tfrac{1}{2}}_{\text{AT}}\cdot\underbrace{\tfrac{1}{2}}_{\text{BT}}\cdot\underbrace{\tfrac{1}{2}}_{\text{AT}}\cdot\underbrace{\tfrac{1}{2}}_{\text{BT}}\cdot\underbrace{\tfrac{1}{2}}_{\text{AT}} + \cdots = \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$