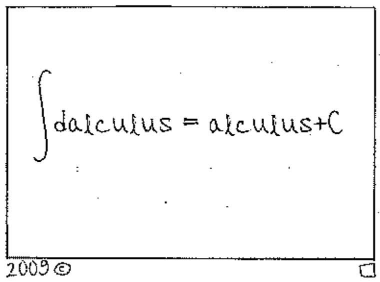
KEY

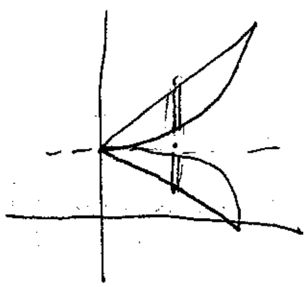
Math 126: Quiz 3

February 25, 2010

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Consider the area bound by $y = x^3 + 1$ and $y = x + 1$ in the first quadrant. Find the volume generated when this area is rotated about the line $y = 1$.



Washers:

$$V = \int \pi \left(r_{out}^2 - r_{in}^2\right) dx$$

$$= \int_0^1 \pi \left(((x+1)-1)^2 - ((x^3+1)-1)^2\right) dx$$

$$= \int_0^1 \pi \left(x^2 - x^6\right) dx$$

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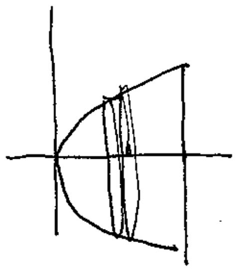
$$= \pi \left. \frac{x^3}{3} - \frac{x^7}{7} \right|_0^1 = \pi\left(\frac{1}{3} - \frac{1}{7}\right) = \pi\left(\frac{4}{21}\right)$$

2. Consider the area bound by $y = 0$, $x = \frac{\pi}{2}$ and $y = \sin(x)$.

(a) Set up (but DON'T EVALUATE) the integral for the volume generated by rotating this area around the x-axis.



$$V = \pi \; y^2 \, dx$$

$$= \pi \int_0^{\pi/2} (\sin x)^2 \, dx$$

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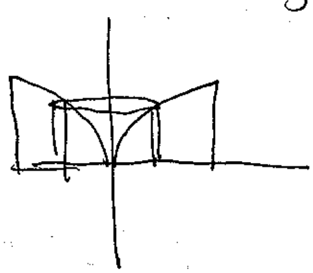
(b) Set up (AND EVALUATE) the integral for the volume generated by rotating this area around the y-axis.

Shells: $V_{shell} = 2\pi r h \, dy = 2\pi x (\sin x) \, dx$



$$2\pi \int_0^{\pi/2} x \sin x \, dx$$

$u = x \qquad dv = \sin x \, dx$

$du = dx \qquad v = -\cos x$

$$2\pi \int_0^{\pi/2} x \sin x \, dx = 2\pi \left[-x\cos x + \int \cos x \, dx \right]$$

$$= 2\pi \left[-x\cos x + \sin x \right]_0^{\pi/2}$$

$$= 2\pi \left[-\frac{\pi}{2}(0) + 1 - 0 + 0 \right]$$

$$= \boxed{2\pi}$$

Do ONE of the following problems.

3. A spring whose natural length is 10 cm requires 40 N of force to hold it at 18 cm. Find the work done in pulling the spring from 10 cm to 20 cm.

OR

4. A 1000 lb. weight is being pulled up to a height of 50 feet by a chain that weighs 4 lb/ft. Find the work done. (Acceleration due to gravity is 32 ft/s^2).

3. $f(x) = kx$

cm beyond length

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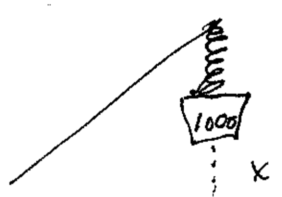
$40 = k(.08)$

$k = \frac{40}{.08} = 500$

$$\text{Work} = \int_0^{0.1} 500x\,dx = 250x^2\Big]_0^{0.1} = 2.5 \text{ J of work.}$$

4]



~~weight~~ mass of system $= 1000 + 4[50 - x]$

$= 1200 - 4x$

$$\text{Work} = \int_0^{50} [1200 - 4x]\,dx$$

$$= 1200x - 2x^2\Big]_0^{50}$$

$= 60000 - 5000 = 55000$ lb ft^2/s^2

(foot pounds)

5. Find

$$\int \arctan(2x)\, dx$$

Int by (silent) parts

$u = \arctan 2x \qquad dv = dx$

$du = \frac{1}{1+4x^2} \cdot 2\, dx \qquad v = x$

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$$\int \arctan(2x)\, dx = x \arctan 2x - \int \frac{2x}{1+4x^2}\, dx$$

$u = 1+4x^2$

$du = 8x\, dx$

$$x \arctan 2x - \frac{1}{4} \ln|1+4x^2| + C$$

6. (Bonus) Comment on the following 'proof'

We solve $\int \frac{1}{x}\, dx$ using integration by parts. Let $u = \frac{1}{x}$ and $dv = dx$. Then $du = \frac{-1}{x^2}\, dx$ and $v = x$. Then:

$$\int \frac{1}{x}\, dx = \frac{1}{x} \cdot x - \int x \cdot \frac{-1}{x^2}\, dx$$

$$\int \frac{1}{x}\, dx = 1 - \int \frac{-1}{x}\, dx$$

$$\int \frac{1}{x}\, dx = 1 + \int \frac{1}{x}\, dx$$

and subtracting the antiderivative from both sides yields...

$$0 = 1$$

poor choice of article. Should be an antiderivative!

As ~~[illegible]~~ antiderivatives are only determined up to a constant, subtracting from both sides may leave that constant behind and it may be different on both sides.