KEY

Math 225: Quiz the First

9.6.06

This exam is closed book and closed notes. You may use your calculator for the purposes of arithmetic and for plotting equations, if helpful. When asked for specific values, however, you must show the relevant algebra. You have until 5 minutes before the hour to finish.

1. Suppose $(x(t), y(t))$ are a set of parametric equations for $a \leq t \leq b$. To reverse the direction of the parametric curve, what algebraic operation do we need to perform?

Replace t with $(-t)$ in both equations and limits.

2. What is the relationship between a circle's radius to a point and its tangent line to that same point?

Radius is perpendicular to tangent.

3. What is the difference between the graph of the line $y = 1 - x$ and the parametric curves $(\cos(t), 1 - \cos(t))$?

$y = 1 - x$ full line, all x & y values

$(\cos(t), 1 - \cos t)$ only values $-1 \leq x \leq 1$ and $0 \leq y \leq 2$.

4. Give the formula for the slope of a tangent line to a polar curve $r = f(\theta)$.

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{r\cos\theta + \frac{dr}{d\theta}\sin\theta}{-r\sin\theta + \frac{dr}{d\theta}\cos\theta}$$

5. Match the following equations $(x(t), y(t))$ to their respective graphs. Give reasons for your choices. Note: There is one extra in each group!

(a) $(\sin(t), \sin(t)\cos(t))$

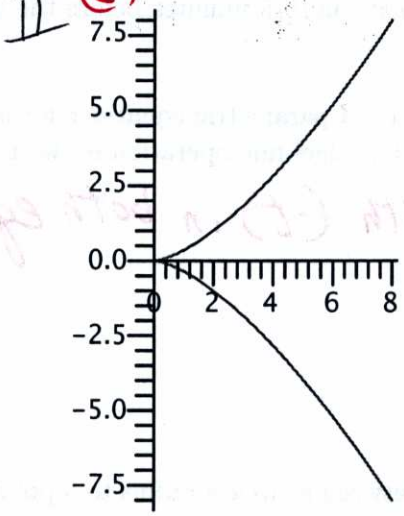
(b) $(\ln(1-t), t)$

out → (c) $(\sin(t), 1-\sin(t))$ (str. line)

(d) $(\sin(4t), \sin(5t))$

(e) $(2t^2, t^3)$

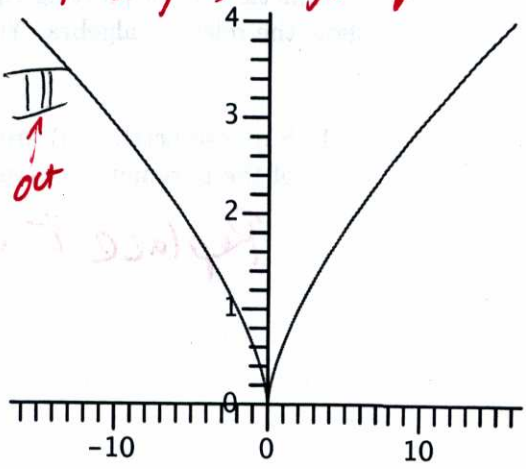
II (e)



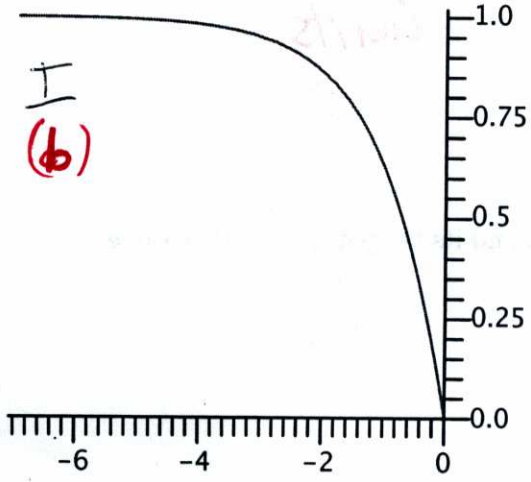
pos, neg x, only pos. y

III

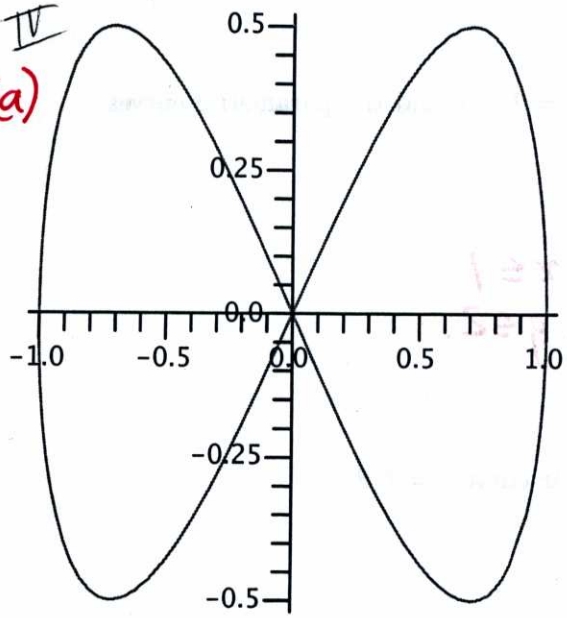
out



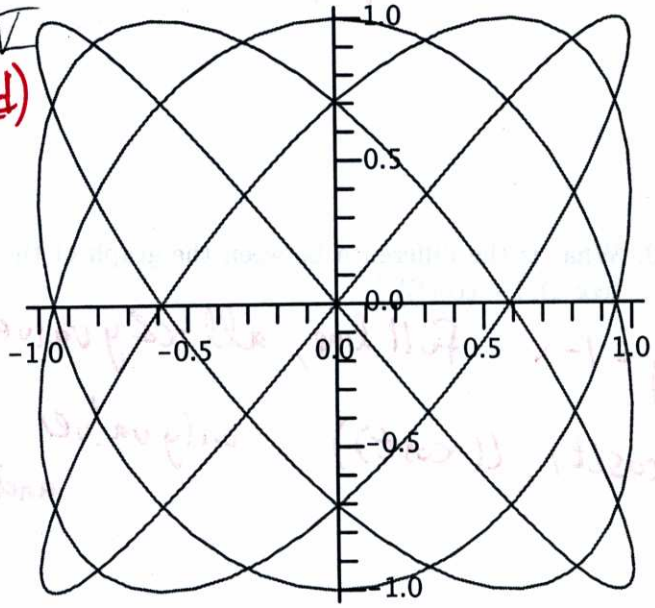
I (b)



IV (a)



V (d)



a IV bounded in x & y, only goes through origin

d V bounded in x & y, lots of x, y intercepts twice per period

e II x positive only, y positive & negative; both unbounded

b I y between 0 & 1, x negative only

~~$x = \ln(1-y)$~~

$\Rightarrow y = 1 - e^x$.

6. Consider the graph of $r = \sin(n\theta)$

(a) How many 'leaves' does this graph have?

n even: $2n$ leaves

n odd: n leaves

(b) Find the total area enclosed by this curve. Note: You should only have to do one integral. How does your answer depend on n?

Single leaf

$0 \le \theta \le \pi/n$

$$\int_0^{\frac{\pi}{n}} \frac{1}{2}(\sin(n\theta))^2 d\theta$$

$$= \int_0^{\frac{\pi}{n}} \frac{1}{4}(1-\cos 2n\theta)\, d\theta$$

$$= \frac{1}{4}\left[\theta - \frac{\sin 2n\theta}{2n}\right]_0^{\pi/n} = \frac{\pi}{4n} - 0 - (0-0)$$

$$= \pi/4n$$

n even $\to 2n$ leaves

total area $= 2n \cdot \pi/4n = \pi/2$

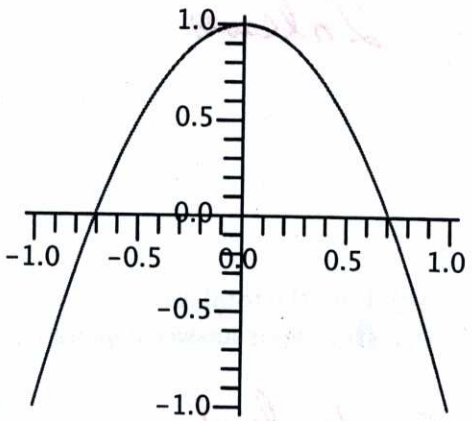
n odd $\to n$ leaves

total area $= n\pi/4n = \pi/4$

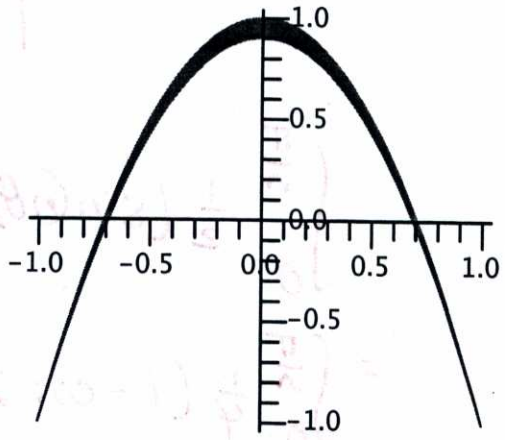
Depends only on parity, not on value.

7. (Bonus) See the graphs below. They represent the graph of the polar equations $x = \sin(t)$ and $y = \cos(2t)$, which we showed in class to be a parabola similar to $y = 1 - 2x^2$, only periodic. The three graphs have increasing bounds on t. Why might we get different pictures for the graphs?

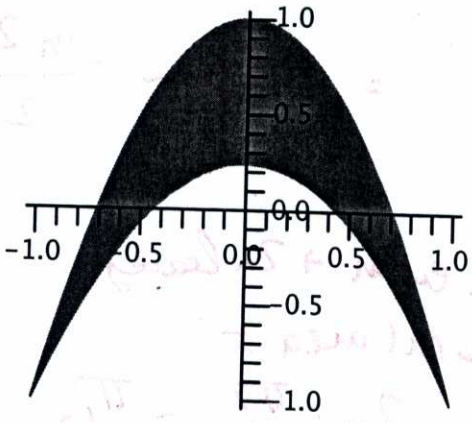
As the interval gets larger, the graph gets more & more "approximate", the points used to plot get farther & farther apart; hence, imperfections increase.



$0 \le t \le 20\pi$



$0 \le t \le 200\pi$



$0 \le t \le 600$