

OCTOBER 12, 2006

Math 225: Quiz the Fifth ~~10/12/2006~~

You know the drill by now. No books, no notes, no colleagues, and no answers without justification.

1. Fill in the Blank

(a) Level curves of a function $z = f(x, y)$ from different heights never (always, sometimes, or never) intersect.

(b) For single variable functions, we only have to take limits from two directions. For multivariable functions, we have to take limits from many many directions.
(infinitely many)

(c) To find $\frac{\partial f}{\partial x}$ of $f(x, y)$, treat y as a constant.

(d) Clairaut's Theorem says that, if f_{xy} and f_{yx} are continuous, then $f_{xy} = f_{yx}$.

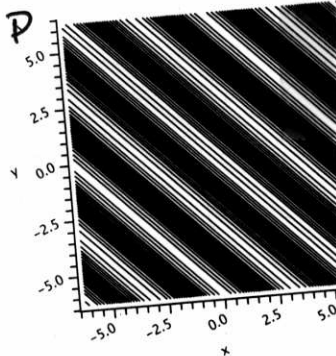
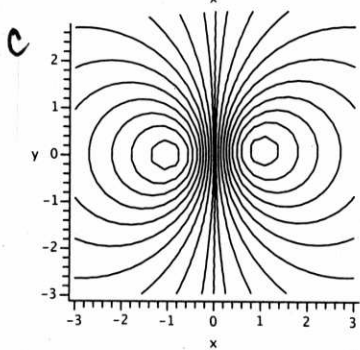
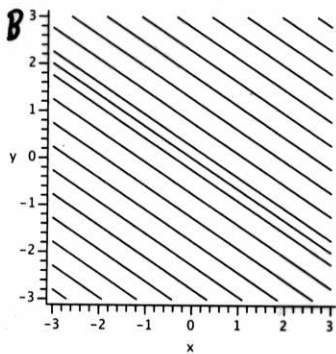
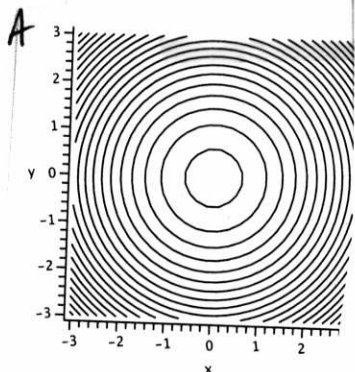
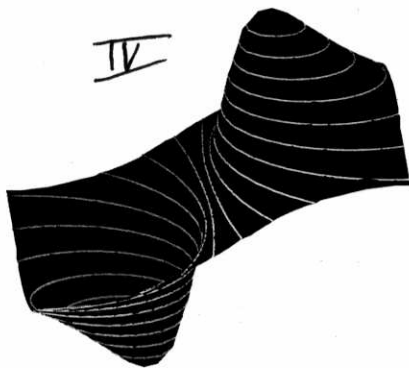
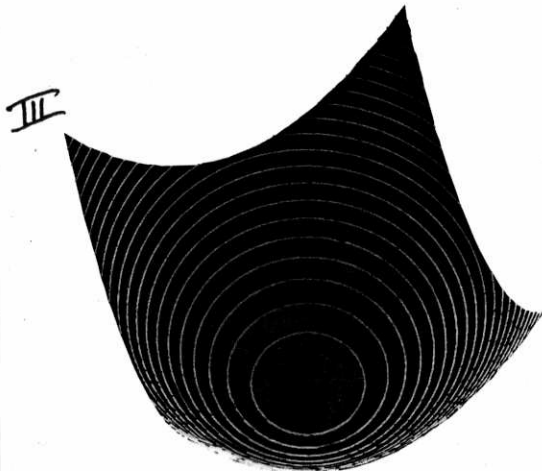
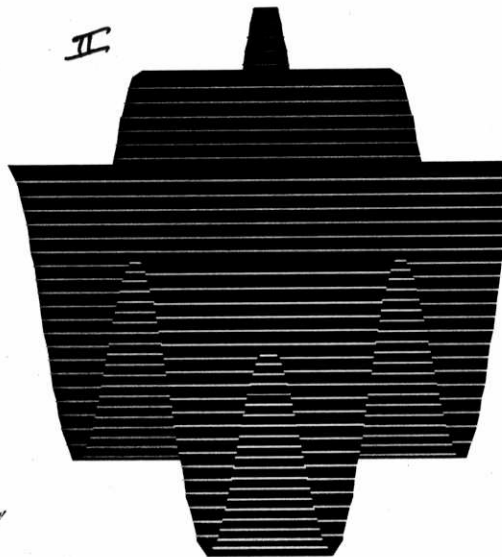
2. Match the equations to the surfaces to the level curve plots. No outliers here.

(a) $f(x, y) = \cos(x + y)$ II, D.

(b) $f(x, y) = x^2 + y^2$ III, A

(c) $f(x, y) = \frac{-5x}{x^2 + y^2 + 1}$ IV, C

(d) $f(x, y) = 2x + 3y$ I, B



3. Find each limit or argue that it doesn't exist.

(a)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{x^2 + 2y^2}$$

$$x=0 \rightarrow L=0$$

$$y=0 \rightarrow L=0$$

$$x=y \rightarrow \frac{2x^2}{x^2 + 2x^2} = \frac{2}{3}$$

$$x=-y \rightarrow \frac{-2x^2}{x^2 + 2x^2} = -\frac{2}{3}$$

LIMIT DNE

(b)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{1+x+y}{1+x^2+y^2}$$

Denominator = $1+x^2+y^2 > 0$, so no issues w/continuity.

$$\text{plugging} \rightarrow \frac{1}{1} = 1.$$

4. Suppose that $f(x, y) = x^2 - 2y^2 + \sin(x - y)$

(a) Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

$$f_x = 2x + \cos(x - y)$$

$$f_y = -4y - \cos(x - y)$$

(b) Find f_{xx} , f_{xy} , f_{yx} and f_{yy} .

$$f_{xx} = 2 - \sin(x - y)$$

$$f_{yy} = -4 + \sin(x - y)$$

$$f_{xy} = f_{yx} = \sin(x - y)$$

(c) Find $f_x(1, 1)$ and $f_y(1, 1)$ and interpret their signs.

$$f_x(1, 1) = 3 > 0 \Rightarrow \text{increase in } x \text{ will increase } f$$

$$f_y(1, 1) = -5 < 0 \Rightarrow \text{increase in } y \text{ will decrease } f$$

5. (Extra Credit) Give an integer between 1 and 100. The lowest number that is not duplicated will receive that percentage extra credit.

Winner: 5