Sixth October 19, 2006

Math 225: Quiz the ~~Fifth~~ ~~March 2, 2005~~

You know the drill by now. No books, no notes, no colleagues, and no answers without justification.
READ ALL QUESTIONS CAREFULLY

1. Fill in the Blank

(a) What is the normal vector to the tangent plane of the surface given by $z = f(x, y)$. You may write out the formula if it helps.

$\vec{n} = \langle f_x, f_y, -1 \rangle$

$z - z_0 = f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$\cancel{f_x(a,b) + f_y(a,b)}$

$f_x(a,b)(x-a) + f_y(a,b)(y-b) - (z - z_0) = 0$

(b) Suppose that $f(x, y, z) = 0$. Find $\frac{\partial z}{\partial x}$. When does this derivative exist?

$\frac{\partial z}{\partial x} = \frac{-f_x}{f_z}$ will exist when f, f_x, f_z are continuous and $f_z \neq 0$

(c) Suppose that f is dependent on x, y and z and each of those is dependent on the same variables, s and t. How many summands are there in $\frac{\partial f}{\partial s}$, ie, how many terms are being added.

3 summands

$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial s}$

(d) Give at least two reasons why the gradient vector is significant.

$\vec{\nabla} f$: • Directional Derivatives

$D_{\vec{u}}(f) = \vec{\nabla} f \cdot \vec{u}$

• $\vec{\nabla} f$ is perpendicular to the level curve / surface

• $\vec{\nabla} f$ is the direction of maximum change

• $\|\vec{\nabla} f\|$ is the rate of max increase.

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2. (a) Find the equation to the tangent plane of $f(x, y) = x^2\sqrt{x^2 + y^2}$ at the point $(5, 12, 325)$

$$z = 325 + f_x(5,12)(x-5) + f_y(5,12)(y-12)$$

$$f_x = 2x\sqrt{x^2+y^2} + x^2 \cdot \frac{x}{\sqrt{x^2+y^2}} \rightarrow 130 + \frac{125}{13}$$

$$f_y = x^2 \cdot \frac{y}{\sqrt{x^2+y^2}} \rightarrow \frac{300}{13}$$

$$z = 325 + \frac{1815}{13}(x-5) + \frac{300}{13}(y-12)$$

(b) Use the plane to approximate the value of f at $(5.02, 11.99)$.

$$f(5.02, 11.99) \approx 325 + \frac{1815}{13}(.02) + \frac{300}{13}(-.01)$$

(c) Does a 'nudge' in x or a 'nudge' in y have a greater effect on the function value near this point?

nudge in x has a greater effect as $f_x > f_y$

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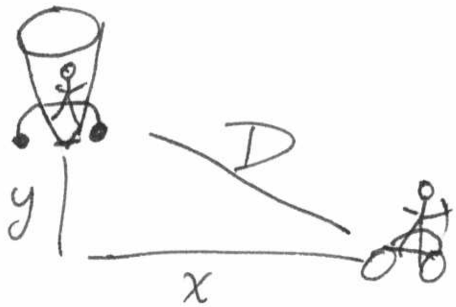
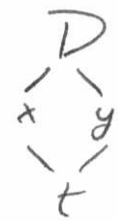
3. A hot air balloon leaves the ground, drifting upward at a rate of 6 ft/s. At the same time, a motorcycle leaves from the same point, speeding off at a rate of 8 ft/s. At what rate is the distance between them changing two seconds after take-off.





$$D = \sqrt{x^2+y^2}$$

$$\frac{dD}{dt} = \frac{\partial D}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial D}{\partial y}\frac{dy}{dt}$$

$t=2$ $x=16$

$y=12$

$$= \frac{x}{\sqrt{x^2+y^2}}\cdot 8 + \frac{y}{\sqrt{x^2+y^2}}\cdot 6$$

$$\Rightarrow \frac{16}{20}\cdot 8 + \frac{12}{20}\cdot 6$$

$$= \frac{32}{5} + \frac{18}{5} = 10\ ft/s.$$

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4. Three ants are on a hot plate whose temperature is given by the function $T(x,y) = 6x^2 + 4xy - 2y^2 + e^{xy}$. The ants are at the point $(0,1)$

(a) Freddy Fire Ant wants to move in the direction of greatest temperature increase. Find that direction, and the rate of increase in that direction.

$$\vec{\nabla} T = \langle T_x, T_y \rangle$$

$$= \langle 12x + 4y + ye^{xy}, 4x - 4y + xe^{xy} \rangle \text{ @ } (0,1)$$

$$= \langle 0+4+1, 0-4+0 \rangle = \langle 5, -4 \rangle \leftarrow \text{direction of greatest temp. increase.}$$

rate of increase = $\sqrt{25+16} = \sqrt{41}$

also acceptable: $\langle \frac{5}{\sqrt{41}}, \frac{-4}{\sqrt{41}} \rangle$

(b) Izzy Ice Ant wants to move in the direction of greatest temperature decrease. Find that direction.

Greatest Decrease $\rightarrow -\vec{\nabla} f$

$\rightarrow \langle -5, 4 \rangle$ or $\langle \frac{-5}{\sqrt{41}}, \frac{4}{\sqrt{41}} \rangle$

(c) Arnie Army Ant wants to move in a direction of no change. Find two such direction vectors.

no change $\rightarrow \vec{u} \perp \vec{\nabla} f$

$\hookrightarrow \vec{v} = \langle 4\ 5 \rangle$

or $\langle -4, -5 \rangle$

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Extra Credit: Tom Swifties (each answer is an adverb).

(a) 'We can use linearization to approximate the function value', Tom said tangentially

(b) 'The angle is more than 90 degrees' Tom said obtusely

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