KEY

Math 225: Quiz the Seventh October 26, 2006

You know the drill by now. No books, no notes, no colleagues, and no answers without justification.
READ ALL QUESTIONS CAREFULLY

1. Find and classify the critical points of $f(x,y) = x^4 + y^4 - 4xy + 2$.

$f_x = 4x^3 - 4y = 0 \rightarrow y = x^3$

$f_y = 4y^3 - 4x = 0 \rightarrow 4x^9 - 4x = 0$

$4x(x^8 - 1) = 0$

$\Rightarrow x = 0, \pm 1$ points $(0,0)$ $(1,1)$ $(-1,-1)$

$y = 0, \pm 1$

$D = f_{xx} f_{yy} - f_{xy}^2$

$= (12x^2)(12y^2) - (4)^2$

$144x^2y^2 - 16$

for $(0,0)$ $D < 0 \rightarrow$ Saddle

$(1,1)$ $D > 0$, max or min $\left\{ f_{xx} > 0 \right.$ so max

$(-1,-1)$ $D > 0$ $f_{xx} > 0$ max

2. What is the maximum value of $f(x, y) = 3x + 2y$ subject to the constraint $x^2 + y^2 = 9$

$f(x,y) = 3x + 2y$ $\quad g(x,y) = x^2 + y^2 = 9$

$f_x = 3 = 2\lambda x = \lambda g_x$

$f_y = 2 = 2\lambda y = \lambda g_y$

$x = \frac{3}{2\lambda} \quad y = \frac{2}{2\lambda}$

$x^2 + y^2 = \frac{9}{4\lambda^2} + \frac{4}{4\lambda^2} = 9$

$\frac{13}{4\lambda^2} = 9 \quad \lambda = \pm\frac{\sqrt{13}}{6}$

we need both $x, y > 0$ to have a max...

$x = \frac{9}{\sqrt{13}} \quad y = \frac{6}{\sqrt{13}}$

max value =

$f(x,y) = \frac{27}{\sqrt{13}} + \frac{12}{\sqrt{13}} = \frac{39}{\sqrt{13}} = 3\sqrt{13}$