KEY

Math 225: Quiz the Ninth
November 29, 2006

You know the drill by now. No books, no notes, no colleagues, and no answers without justification. PLEASE READ ALL OF THE QUESTIONS CAREFULLY

1. Fill in the blank

   (a) There are <u>six</u> different, but equivalent, 'orders' in which to write a triple integral.

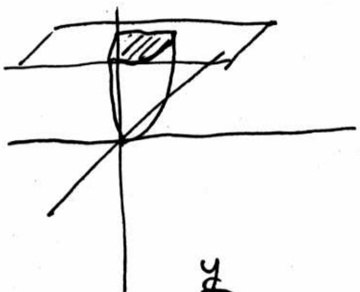
   (b) $\iiint f(x,y,z)\,dz\,dy\,dx = \iiint$ <u>$f(r\cos\theta, r\sin\theta, z)\, r$</u> $dz\,dr\,d\theta$

   (c) $\iiint f(x,y,z)\,dz\,dy\,dx = \iiint$ <u>$f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\,\rho^2\sin\phi$</u> $d\rho\,d\phi\,d\theta$

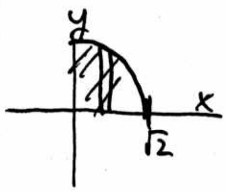
   (d) $\iint f(x,y)\,dy\,dx = \iint f(x(u,v), y(u,v))$ <u>$\left|\begin{matrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{matrix}\right|$</u> $du\,dv$ (give both a formula and a name for this one)

   Jacobian

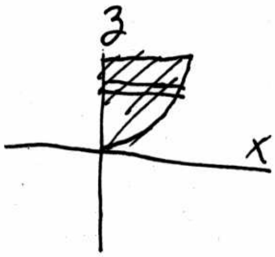
2. Set up TWO DIFFERENT triple integrals in rectangular (xyz) coordinates to find the volume bound by the planes $x = 0, y = 0, z = 2$ and the paraboloid $z = x^2 + y^2$. Each of your DIFFERENT integrals should have a DIFFERENT variable as your innermost variable. DO NOT ATTEMPT TO EVALUATE THESE INTEGRALS.



$$\int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}}\int_{x^2+y^2}^{2} dz\,dy\,dx$$



OR

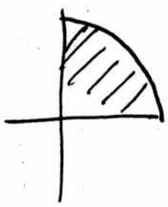


$$\int_0^{2}\int_0^{\sqrt{z}}\int_0^{\sqrt{z-x^2}} dy\,dx\,dz$$

or others.

$$\int_0^{\sqrt{2}}\int_{x^2}^{2}\int_0^{\sqrt{z-x^2}} dy\,dz\,dx$$

3. Find the volume in question 2 by setting up an integral in cylindrical coordinates.



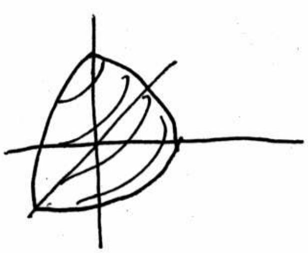
$$\int_0^{\sqrt{2}}\int_0^{\pi/2}\int_{r^2}^{2} r \, dz \, d\theta \, dr$$

$$= \int_0^{\sqrt{2}}\int_0^{\pi/2} 2r - r^3 \, d\theta \, dr$$

$$= \frac{\pi}{2}\int_0^{\sqrt{2}} 2r - r^3 \, dr = \frac{\pi}{2}\left(r^2 - \frac{r^4}{4}\right]_0^{\sqrt{2}}$$

$$= \frac{\pi}{2}\left(2 - \frac{4}{4}\right) = \boxed{\frac{\pi}{2}}$$

4. Find the volume of the sphere of radius 1 centered at the origin that lies in the first octant, using an integral in SPHERICAL coordinates.



$0 \le \rho \le 1$

$0 \le \phi \le \frac{\pi}{2}$

$0 \le \theta \le \pi/2$

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{1} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{\pi/2}\int_0^{\pi/2} \frac{1}{3}\rho^3\Big]_0^1 \sin\phi \, d\phi \, d\theta$$

$$= \int_0^{\pi/2}\int_0^{\pi/2} \frac{1}{3} \sin\phi \, d\phi \, d\theta$$

$$= \int_0^{\pi/2} -\frac{1}{3}\cos\phi\Big]_0^{\pi/2} d\theta = \int_0^{\pi/2} -\frac{1}{3}(0-1)\, d\theta$$

$$= \int_0^{\pi/2} \frac{1}{3} d\theta = \boxed{\frac{\pi}{6}} = \frac{\frac{4}{3}\pi}{8}$$

5. Determine

$$\iint_R (2x+y)e^{x-y}\,dx\,dy$$

where R is bound by the lines $2x+y=1$, $2x+y=3$, $x-y=-3$ and $x-y=3$.

$u = 2x+y \leftrightarrow x = \frac{u+v}{3}$

$v = x-y \leftrightarrow y = \frac{u-2v}{3}$

$R \to \begin{matrix} 1 \le u \le 3 \\ -3 \le v \le 3 \end{matrix}$

$u = 2x+y$

$2v = 2x-2y$

Jacobian $= \left| \begin{matrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-2}{3} \end{matrix} \right| = \left| \frac{-2}{9} - \frac{1}{9} \right| = \frac{1}{3}$

$\frac{1}{3} \int_{-3}^{3} \int_{1}^{3} u e^{v}\, du\, dv$

$= \frac{1}{3} \int_{-3}^{3} \left. \frac{u^2}{2} \right]_1^3 e^v\, dv$

$= \frac{1}{3} \int_{-3}^{3} 4e^{v}\, dv$

$= \frac{4}{3}(e^{3} - e^{-3})$

EXTRA CREDIT: I have an urn with 90 balls. 30 are red. 60 are black or yellow, but I have no idea how many of each there are, just that all distributions are equally likely.

Would people rather play game A or Game B, and why? What about for yourself?

(A) Pull a ball, win if it's red. (B) Pull a ball, win if it's black. or A) R or Y B) B or Y

People would rather play A, People would rather play B.