Math 225: Exam the First

You have 90 minutes to complete this test. This test is closed book, closed notes, and closed colleagues. You may use your calculator for the purposes of graphing equations, but be prepared to justify your answers algebraically.

1. Let $x = \ln(t)$ and $y = e^t$ be parametric equations.
   (a) What is the permissible range of t values? What values of x and y are determined by these equations?
   (b) Determine $\frac{dy}{dx}$ and use it to determine the equation of the tangent line at $(\ln(2), e^2)$.
   (c) Determine the area bound by the curve, the axes, and the line $x = -4$ (Leave your answer as an integral in t).
   (d) Eliminate the parameter in the equations and verify your answers to parts b and c.
2. Plot the polar curve $r = \frac{3}{2} + cos(\theta)$ for $-\pi < \theta < \pi$.
   (a) Does this curve ever go through the origin? Why or why not?
   (b) Determine, algebraically, the values of θ for which the curve has vertical tangents.
   (c) Set up the integral which gives the area enclosed by this curve in the third quadrant. Describe how to solve this integral, but don't actually carry out the calculations.
3. (a) Describe, using inequalities and rectangular coordinates, the points in the first octant outside of the sphere of radius 2 centered at the origin.
   (b) Same as part (a), but with spherical coordinates.
4. Argue (using vectors) that a parallelogram is a rectangle if and only if its diagonals are equal in length.
5. Consider the points $P = (1, 3, -2), Q = (3, 1, 6)$, and $R = (1, 2, -1)$
   (a) Find the perimeter of triangle PQR

(b) Is this triangle right, acute, or obtuse? (recall that an angle is obtuse if it is greater than $\frac{\pi}{2}$)?

(c) Find the equation of the plane that contains this triangle.

6. Below are a set of several equations in various 3 dimensional coordinate systems. Which of these equations represent the same surfaces? (By same, I mean the exact same surface in the same location). There are 9 equations, with three pairs and one triplet, each with the same surface. ALSO: Describe each surface in words.

   (a) $x^2 + y^2 - 9z^2 = 1$

   (b) $y = 3$

   (c) $x = y$

   (d) $\rho^2(\sin^2(\phi) - 9\cos^2\phi) = 1$

   (e) $r = 3\csc(\theta)$

   (f) $\rho = 3\cos(\phi)$

   (g) $\rho = 3\csc(\phi)\csc(\theta)$

   (h) $\theta = \frac{\pi}{4}$

   (i) $x^2 + y^2 + (z - \frac{3}{2})^2 = \frac{9}{4}$

7. Consider the elliptical helix given by the vector valued function $\mathbf{r}(t) = \langle 3\cos(t), 2\sin(t), t\rangle$.

   (a) Find the tangent line to this curve when $t = \frac{\pi}{2}$

   (b) Find the angle between this curve and the curve $\mathbf{s}(\mathbf{t}) = \langle t - 3, t^2, t^3 + \pi\rangle$ at the point $(-3, 0, \pi)$. WARNING: There are different t values at this point on the two curves.