Math 225: Practice Exam the Second

1. Draw the level curves of the function $f(x, y) = xy$ for $f(x, y) = -2, -1, 0, 1, 2$.
   (a) Do the curves intersect each other?
   (b) What kind of critical point is (0,0)? What does the level curve look like there? Can you make a statement about such critical points?

2. Find
$$\lim_{(x,y)\to(0,0)} \frac{(x-y)^2}{x^2+y^2}$$
or explain why it doesn't exist.

3. Suppose that a continuous and differentiable function $f(x, y)$ has $\frac{\partial f}{\partial y} = x^2y + 3y^2$ Which of the following are possible results for $\frac{\partial f}{\partial x}$?
   (a) $xy^2 + 2x$
   (b) $3xy + x^2$
   (c) $xy^2 + \cos(x)$
   (d) $x^2y + 3x^2$

4. By about how much will $f(x, y) = \cos(\pi xy) + x^2y^3$ change if we move from the point $(-1, -1)$ to the point $(-.95, -.98)$? Use the tangent plane for your estimate.

5. Suppose that an engineer wants to build a railroad track up the mountain whose equation is given by $f(x, y) = 2800 - 0.03x^2 - 0.05y^2$ from the point $(-1, 2)$.
   (a) In what direction should he build his track if he wants it to ascend the fastest?
   (b) What is the angle of ascent for this track (The angle of ascent, in radians, is $\tan(\theta) = |\nabla(f)|$)? Is this above or below the maximum allowable grade of 5°? (WATCH YOUR UNITS)
   (c) In how many directions may we build the train to achieve the maximum allowable grade. (The grade θ in a direction $\mathbf{u}$ is given by $\tan(\theta) = |D_{\mathbf{u}}(f)|$
   (d) (Bonus) Find the directions that give the maximum allowable grade. (And in so doing, maximize your grade!)

6. Student performance at Whitman has been determined by the amount S of sleep a student receives, as well as the amount C of caffeine a student takes in. Learning can be modeled by $L(S, C) = 3S^2 + 2C^2 + 4C$. However, sleep and caffeine interfere with each other, thus any student is constrained in their intake of both by $S^2 + C^2 = 10$. How much more important is one than the other?

7. Set up
$$\iint_R e^{y^3}\, dA$$
two ways. R is bound by the y axis, the line $y = 1$, and the curve $y = \sqrt{x}$. Determine which of the integrals is solvable and solve it.