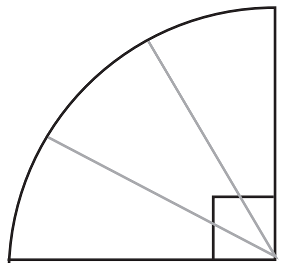
Math 225: Final Examination
May 17, 2005

You may use a calculator on this exam. Please keep your work neat and justify your answers appropriately. Please **READ EACH QUESTION CAREFULLY** before answering. Don't spend too long on any one question. See me if you have any issues.

1. Consider the circle of radius 1 centered at the point (1,0).
   (a) Find a parametrization for this circle
   (b) Find a polar equation for this circle
   (c) Verify (using any integral you wish) the circumference of this circle
   (d) Verify (using any integral you wish) the area of this circle.
2. Consider $A = (1, 2, 3), B = (-1, 0, 4), C = (2, -3, 5)$
   (a) Find the equations of the lines AB and AC and the angle between these lines.
   (b) Find the equation of the plane containing AB and AC.
   (c) Find the equation of the line normal to the plane passing through the point C.
3. Consider the curve with parametric equation $\mathbf{r}(t) = \langle t^2, t, t^3 \rangle$.
   (a) Find the tangent line to this curve at the point when $t = 2$.
   (b) Find the arc length of this curve from $t = 0$ to $t = 2$. Leave your answer as an integral.
   (c) Does this curve have tangent, normal, and binormal vectors at the origin? Explain.
4. Let $f(x, y) = x^2 + y^4 + e^{xy}$. Approximate $f(1.02, -.01)$.
5. What is the minimum distance between the curve $xy = 1$ and the origin?
6. Draw the level curves of the function $f(x, y) = y \csc x$. Do these level curves intersect? Why or why not?
7. (a) Determine the volume of a cylinder of radius R and height h. Prove your answer is correct by solving the appropriate integral.
   (b) Find the volume bound by the cylinder $x^2 + y^2 = R^2$, the xy-plane, and the plane $z = ax + by + h$ and comment on your answer. (This may help you with part (a)).
8. Use an integral to find the volume of the tetrahedron formed by the coordinate planes and the plane $ax + by + cz = d$ (a,b,c,d positive), and verify that this volume is equal to $\frac{1}{3}Bh$, where B is the area of the base, and h is the height.
9. (a) Find the center of mass of the plate whose vertices are $(-1, 0), (1, 2)$ and $(-1, 2)$, and whose density is given by $\rho(x, y) = x^3 + x^2y^2$.
   OR

(b) Find the probability of hitting the ball to right field in the picture of the baseball diamond presented below, given that all points are equally likely. (Each of the three segments of the arc are equal to the others).



10. Deternine

$$\iiint_E e^{(x^2+y^2+z^2)^{\frac{3}{2}}}\ dV$$

where E half of the unit sphere above the xy-plane.

11. Determine

$$\iint_R x^2 - y^2\ dA$$

where R is the square with vertices $(1, 0), (0, 1), (-1, 0)$, and $(0, -1)$ by making an appropriate change of coordinates.

How does the region of integration after the change compare with the given region?

12. Determine the following integrals. Use any theorems or methods that are appropriate.

(a)

$$\int_C (2xe^y)\ dx + (2y + x^2e^y)\ dy$$

where C is the eighth of the unit circle from $(1, 0)$ to $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$.

(b)

$$\int_C x^2y\ dx + y^2\ dy$$

where C is the line segment from $(0, 0)$ to $(2, 3)$

(c)

$$\int_C (2x + 2y + \cos^3(x))\ dx + (\ln(\tan(y)) + e^{y^2} + x^2)\ dy$$

where C is the closed square from $(0, 0)$ to $(1, 0)$ to $(1, 1)$ to $(0, 1)$.

(d)

$$\oint_C \ln x + 2y\ dx + \sin(y)e^{\cos(y)} + x^2\ dy$$

where C is the right half of the unit circle oriented counterclockwise.

13. List as many (at least 3) integral theorems as you can and draw any parallels that you can between them. Note: I want *theorems* and not *formulas*.

14. Choose a concept from single-variable calculus that was revisited in multivariable calculus and explain the parallels and the differences between the concept in the two contexts. Write your answer to someone who is finishing Calculus II and is thinking about taking Calculus III.