Math 225: Exam the First

Spring 2008

You have two hours to complete this exam. You may use a calculator for computation only, and you should be prepared to show the relevant steps to a problem where necessary.

1. (a) Give the equation, in rectangular coordinates, of the cylinder of radius 6 centered around the z-axis.
   (b) Give the equation, in rectangular coordinates, of the cylinder of radius 6 centered around the y-axis.
   (c) Find, in parametric form, the equation of the curve of intersection of the cylinder in part (b) with the plane $y + 4z = 3$, and describe the curve.
2. Let ℓ_1 be the line through the two points (-3, 1, 0) and (1,1,2), and ℓ_2 be the line through the points (6, 2, 6) and (3, -1, 0).
   (a) Find the point of intersection of ℓ_1 and ℓ_2.
   (b) Find the plane that contains both lines.
3. (a) The equation in spherical coordinates:
   $$\rho = 4 \sin\phi(\cos\theta + \sin\theta)$$
   defines a sphere. Find its center and radius. (Hint: Multiply both sides by ρ and convert to rectangular coordinates).
   (b) Convert the center point to cylindrical and spherical coordinates.
4. (a) Find the equation of the tangent line to the curve $\mathbf{r}(t) = \langle 4 - t, 3t - t^2, t\rangle$ at the point when $t = 0$.
   (b) Using your work in part (a), find $\mathbf{T}(0)$. (Do NOT try to calculate a generic formula for $\mathbf{T}(t)$).
   (c) We can show that $\mathbf{T}'(0) = \langle -3, -2, 3\rangle$. Find $\mathbf{N}(0)$, $\mathbf{B}(0)$, and the osculating plane to the curve at $t = 0$.
5. Show that if vectors $\mathbf{x} - \mathbf{y}$ and $\mathbf{x} + \mathbf{y}$ are orthogonal, then $\mathbf{x}$ and $\mathbf{y}$ must have the same length.
6. Let $\mathbf{x}$ and $\mathbf{y}$ be unit vectors. What are the minimum and the maximum magnitude of $\mathbf{x} \times \mathbf{y}$, and what is the geometric relationship between $\mathbf{x}$ and $\mathbf{y}$ when these are achieved? Why?
7. Suppose that a particle is moving with acceleration
   $$\mathbf{a}(t) = \langle 6t, \cos(t), e^t\rangle$$
   and that the object starts with initial velocity vector $\langle 2, 1, 2\rangle$ and initial position vector $\langle 0, 1, 3\rangle$. Find the position of the object when $t = 1$.
8. (Extra Credit) Prove that a straight line has zero curvature.