Math 225: Final Exam

Tuesday, May 13, 2008

You have 150 minutes to complete this closed-book, closed-notes, and closed-colleague exam. You may use a calculator, but be prepared to justify your answers with the relevant algebra or arguments.

READ ALL QUESTIONS CAREFULLY

1. Consider the triangle ABC where

$$A = (1, 1, 2), B = (2, 3, 4), C = (3, 4, 6)$$

   (a) Calculate $\mathbf{x} = \overrightarrow{AC}, \mathbf{y} = \overrightarrow{AB}$ use them to find the plane that contains the triangle ABC.

   (b) Find the area of triangle ABC (Hint: It's half of a certain parallelogram).

   (c) Calculate $\mathbf{z} = \text{proj}_{\mathbf{y}}\mathbf{x}$

   (d) Calculate $\frac{1}{2} \cdot |\mathbf{x}| \cdot |\mathbf{y} - \mathbf{z}|$ and explain why your answer makes sense.

2. Let $\mathbf{r}(t) = \langle \cos(t), \sin(t), \cos(2t) \rangle$ describe the path of a particle in 3-space.

   (a) What are the permissible x, y and z values for $\mathbf{r}(t)$

   (b) Is this curve smooth? Why or why not?

   (c) This curve is the intersection of which two surfaces?

3. Let

$$f(x, y) = \frac{1}{x^2 + y^2 + 1}$$

   (a) Is f defined everywhere? Explain.

   (b) For which values of k does $f(x, y) = k$ produce level curves? What do these curves look like?

   (c) Find the equation of the tangent plane to $f(x, y)$ at the point $(2, -1)$. Use it to approximate $f(1.97, -.97)$.

4. Find the minimum value of $f(x, y, z) = x^2+y^2+z^2$ subject to the constraint $2x+6y+10z = 140$

5. Compute

$$\int_0^1 \int_{\sqrt{x}}^{\sqrt[3]{x}} xy \, dy \, dx.$$

   Then, reverse the order of integration and verify your answer.

6. Consider a plate in the right half of the xy-plane bound by the curves $x^2 + y^2 = 1$ and $x = 1 - y^2$. The density of the plate is given by the function $\rho(x, y) = x$.

   (a) Find the mass of the plate.

   (b) Find $\overline{y}$ for the plate. (Hint: You may argue rather than compute here)

(c) Set up the integral to compute $\overline{x}$. Do not attempt to compute your integral.

(d) Is the center of mass actually on the plate? If not, explain how this can be.

7. Compute

$$\iiint_E \cos((x^2+y^2+z^2)^{\frac{3}{2}})\ dV$$

where E is the portion of the unit sphere centered at the origin in the first octant.

8. Set up an integral to determine the length of one period of $y = \sin(x)$. Do not attempt to compute your integral.

9. Find the volume contained within the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

10. Compute the following integrals.

(a)

$$\int_C x\ ds$$

where C lies on the parabolic curve $y = 2x^2 + 3$ from $(1, 5)$ to $(3, 21)$.

(b)

$$\int_C (x^2 + 2xy)\ dx + (x^2 + y^2)\ dy$$

where C is the curve $\langle t^2 \sin(t), \cos(t)\rangle$ from $0 \leq t \leq \frac{\pi}{2}$

(c)

$$\oint_C (e^x + 2y)\ dx + (\arctan(y) + x^2)\ dx$$

where C is the parabolic segment along $y = x^2$ from $(0, 0)$ to $(2, 4)$ followed by the line segment from $(2, 4)$ to $(0, 0)$.

11. List as many (at least 3) integral theorems as you can and draw any parallels that you can between them. Note: I want *theorems* and not *formulas* (ie, 'The integral for arclength' is not a *theorem*).

12. Choose a concept from single-variable calculus that was revisited in multivariable calculus and explain the parallels and the differences between the concept in the two contexts. Write your answer to someone who is finishing Calculus II and is thinking about taking Calculus III.