Key

Math 225: Quiz the First

This exam is closed book and closed notes. You may use your calculator for the purposes of arithmetic operations (including trig). When asked for specific values, however, you must show the relevant algebra. You have 40 minutes.

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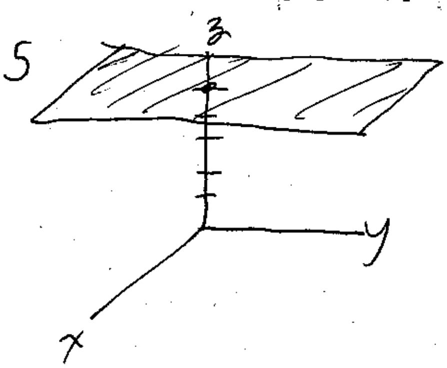
1. Give an algebraic equation for each of the following surfaces. Note: You may draw each surface if you think that would be helpful.

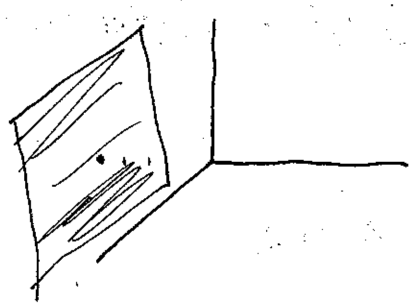
   (a) A plane five units above the xy- plane, parallel to the xy-plane.

   $z = 5$



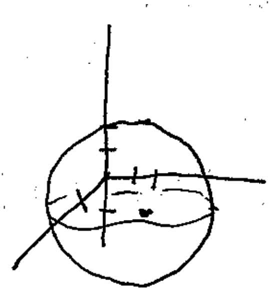
   (b) A plane parallel to the xz-plane, and four units to its left.

   $y = -4$



   (c) A sphere of radius 3 centered at the point (1, 2, -1).

   $(x-1)^2 + (y-2)^2 + (z+1)^2 = 9$



2. Find the center and radius of the sphere:

$$x^2 + y^2 + z^2 - 6x + 4y - 10z = 43$$

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$$x^2 - 6x + y^2 + 4y + z^2 - 10z = 43$$
$$+9 \qquad +4 \qquad +25 \qquad +38$$

$$(x-3)^2 + (y+2)^2 + (z-5)^2 = 81$$

$c: (3, -2, 5) \qquad r = 9$

3. Given points $A = (2, 1, 0)$, $B = (1, -1, 1)$ and $C = (3, -3, 2)$, calculate

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(a) The distance between B and each of the other two points

(b) The angle formed at B (Use vectors. You may leave your answer as an inverse-trig value)

a) $d(B, A) = \sqrt{(1-2)^2 + (-1-1)^2 + (1-0)^2} = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

$d(B, C) = \sqrt{(1-3)^2 + (-1-3)^2 + (1-2)^2} = \sqrt{4+4+1} = 3$

b) $\overrightarrow{BA} = \cancel{\langle 1, 2, 1 \rangle} \langle 1, 2, -1 \rangle$

$\overrightarrow{BC} = \langle +2, -2, 1 \rangle$

$\overrightarrow{BA} \cdot \overrightarrow{BC} = 1 \cdot 2 + (-2) 2 + (-1) \cdot 1$

$= -3$

$\theta = \arccos\left(\frac{-3}{\sqrt{6} \cdot 3}\right) = \arccos\left(\frac{-1}{\sqrt{6}}\right)$

4. Suppose that **x**, **y**, and **z** are vectors with their heads on points of the same circle (and their tails at the center of the circle. Suppose further that **x** and **y** point in opposite directions.

(a) What is are the relative magnitudes of **x**, **y** and **z**?

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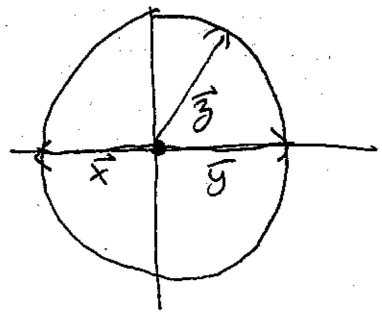
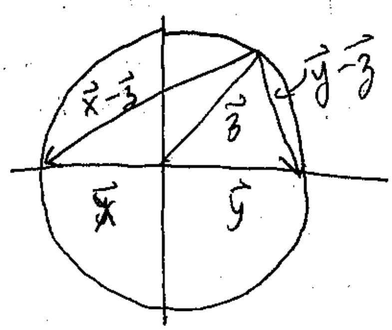
$|\vec{x}| = |\vec{y}| = |\vec{z}|$ = radius of the circle



$\vec{x} = -\vec{y}$

(b) Prove that $\mathbf{x} - \mathbf{z}$ is orthogonal to $\mathbf{y} - \mathbf{z}$.



$$(\vec{x}-\vec{z})\cdot(\vec{y}-\vec{z})$$
$$= (-\vec{y}-\vec{z})\cdot(\vec{y}-\vec{z})$$
$$= -\vec{y}\cdot\vec{y} - \vec{z}\cdot\vec{y} + \vec{z}\cdot\vec{y} + \vec{z}\cdot\vec{z}$$
$$= \vec{z}\cdot\vec{z} - \vec{y}\cdot\vec{y}$$
$$= |\vec{z}|^2 - |\vec{y}|^2 = 0 \text{, since } |\vec{y}| = |\vec{z}|$$

V, ☺, ek.

(Hint: DRAW A PICTURE)

5. Puxatawney Pete (Phil's autumnal cousin) is preparing for his pending prediction. When he pops out of his hole, his body forms the vector $\mathbf{x} = \langle 4, 2, 3 \rangle$ and his shadow forms the vector $\mathbf{y} = \langle 1, 2, 0 \rangle$.

(a) Project the 'body' vector onto the 'shadow' vector.

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$$\text{proj}_{\vec{y}} \vec{x} = \frac{\vec{x} \cdot \vec{y}}{|y|^2} \vec{y} = \frac{\langle 4,2,3\rangle \cdot \langle 1,2,0\rangle}{|\langle 1,2,0\rangle|^2} \langle 1,2,0 \rangle$$

$$= \frac{8}{5} \langle 1,2,0\rangle = \langle \frac{8}{5}, \frac{16}{5}, 0 \rangle$$

$$\cancel{= \langle \frac{4}{3}, \frac{8}{3}, 0 \rangle}$$

$$\langle \frac{8}{5}, \frac{16}{5}, 0 \rangle$$

(b) Is the projection longer, shorter, or the same length as the shadow vector? Is the projection longer, shorter, or the same length as the body vector?

projection is $\frac{8}{5}$ as long as shadow vector, longer

$$|\text{proj}| = \cancel{\sqrt{\frac{16}{9} + \frac{64}{9} + 0} = \frac{\sqrt{80}}{3}} \quad \sqrt{\frac{64}{25} + \frac{256}{25}} = \sqrt{\frac{320}{25}} = \frac{8}{\sqrt{5}}.$$

$$|\text{body}| = \sqrt{16+4+9} = \sqrt{29}$$ & body is longer

(c) (Bonus) Would the body vector always be longer than the projection? Why or why not?

Body is longer in general

$$|\text{proj}_{\vec{y}} \vec{x}| = \frac{\vec{x} \cdot \vec{y}}{|\vec{y}|} = \frac{|\vec{x}| \cdot |\vec{y}| \cos\theta}{|\vec{y}|} = |\vec{x}| \cos\theta \le |\vec{x}|.$$