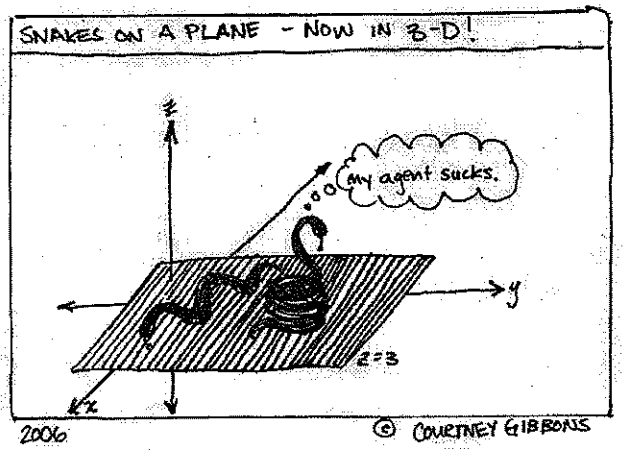
Key

Math 225: Quiz the Second

This quiz is closed book and closed notes. You may use your calculator for the purposes of arithmetic and for plotting equations, if helpful. When asked for specific values, however, you must show the relevant algebra. PLEASE READ ALL QUESTIONS CAREFULLY. You have 40 minutes.



1. Let $\mathbf{x}, \mathbf{y}$ and $\mathbf{z}$ be vectors in $\mathbb{R}^3$. For each quantity listed, tell whether it is a vector, a scalar, or if the quantity does not make sense.

5

(a) $\frac{1}{|\mathbf{x}|}(\mathbf{y} \cdot \mathbf{z})$ Scalar

(b) $(\mathbf{x} - \mathbf{y}) \times \mathbf{z}$ vector

(c) $(\mathbf{x} \cdot \mathbf{y}) + \mathbf{z}$ nonsense

(d) $(\mathbf{x} \times \mathbf{z}) \times (\mathbf{y} \times \mathbf{z})$ vector

(e) $\mathbf{x}^2$ nonsense (need to specify dot, cross, magnitude)

2. Find a unit vector perpendicular to $\langle 1, 3, 1\rangle$ and $\langle -2, 4, 2\rangle$.

3 $\langle 1, 3, 1 \rangle$

$\times \langle -2, 4, 2 \rangle$

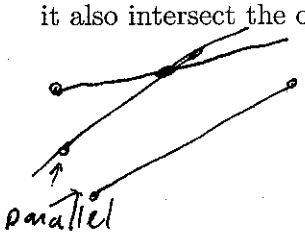
$\langle 6-4, -2-2, 4-(-6) \rangle$

$= \langle 2, -4, 10 \rangle$ ← perp to both, mag $= \sqrt{120}$

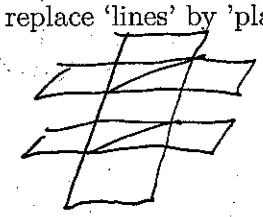
$\rightarrow \left\langle \frac{2}{\sqrt{120}}, \frac{-4}{\sqrt{120}}, \frac{10}{\sqrt{120}} \right\rangle$ unit, perp to both

3. Suppose that we have two parallel lines, and that a third line intersects one of the two. Must it also intersect the other? Why or why not? What if we replace 'lines' by 'planes'?

3



No, the two nonintersecting lines could be skew.



Yes, planes cannot be skew, they either intersect or are parallel

4. Find the plane perpendicular to the line $\frac{x-1}{2} = 1 - y = \frac{z-3}{4}$ and containing the point (2,2,-1).

4 For a plane we need a

point (given, (2,2,-1))

& a normal vector (from the line: $\langle 2,-1,4 \rangle$)

Plane: $2(x-2) - 1(y-2) + 4(z+1) = 0$

or $2x - y + 4z = -2$

5. Find the line parallel to $\frac{x-1}{2} = 1 - y = \frac{z-3}{4}$ through the point (1, 2, 3).

4 For a line we need a

point (1,2,3)

& a direction vector

(from ~~the~~ line $\langle 2,-1,4 \rangle$)

line: $\vec{r}(t) = \langle 1,2,3 \rangle + t\langle 2,-1,4 \rangle$

6. (a) Find the line of intersection of the two planes

$$x + y + z = 3; x + 2y + 4z = 7$$

4 For the line we need a
point (by inspection (1,1,1))
& a direction vector
perp to both normals, so

$\langle 1,1,1\rangle \times \langle 1,2,4\rangle = \langle 2,-3,1\rangle$

line: $x = 1+2t$, $y = 1-3t$, $z = 1+t$

(b) Does your line in part (a) intersect the line

$$\langle x, y, z\rangle = \langle 1,0,3\rangle + t\langle -2,2,1\rangle$$

2 and, if so, where?

$x = 1-2t = 1+2s$

$y = 2t = 1-3s$

$z = 3+t = 1+s$

y, z give $t = \frac{1-3s}{2}$

$3 + \left(\frac{1-3s}{2}\right) = 1+s$

$\frac{7}{2} - \frac{3}{2}s = 1+s$

$\frac{5}{2}s = \frac{5}{2}$; $s = 1$

$t = \frac{1-3(1)}{2} = -1$

and for x, $1-2(-1) = 1+2(1)$ ✓
so they do intersect.

$x = 1+2(1)$, $y = 1-3(1)$; $z = 1+1$

Intersect @ $(3,-2,2)$

Extra Credit: Would you like 1 point or 3 points extra credit? Note: If more than 25% of you choose 3 points, no one gets anything.

My guess: 45% will opt for 3.