Key

Math 225: Quiz the Third

9/26/08

This exam is closed book and closed notes. You may use your calculator for the purposes of arithmetic and for plotting equations, if helpful. When asked for specific values, however, you must show the relevant algebra. You have 40 minutes.

1. Convert the following equations as requested.

   (a) $x^2 + y^2 = z^2 - 1$, from rectangular coordinates to cylindrical coordinates.

   $r^2 = z^2 - 1$

   or $z^2 - r^2 = 1$

   (b) $x = y$, from rectangular coordinates to cylindrical coordinates.

   $x = y \Rightarrow r\cos\theta = r\sin\theta$

   $\cos\theta = \sin\theta$

   $\tan\theta = 1, \ \theta = \pi/4$

   (c) $\rho = 6\cos(\phi)$, from spherical coordinates to rectangular coordinates (Hint: Multiply both sides by ρ).

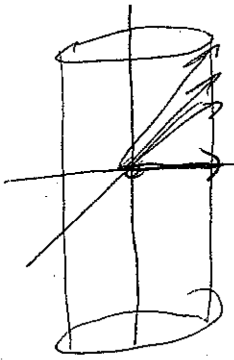
   $\rho^2 = 6\rho\cos\phi$

   $x^2 + y^2 + z^2 = 6z$

   or $x^2 + y^2 + z^2 - 6z = 0$

   or $x^2 + y^2 + (z-3)^2 = 9$

2. On the homework you showed that rectangular equation for the cylinder $x^2 + y^2 = 4$ converts to the spherical equation $\rho \sin \phi = 2$. Investigate what happens to ρ as ϕ decreases from $\frac{\pi}{2}$ to 0, and use it to help explain the shape of the cylinder.



$\phi = \pi/2, \rho = 2$

As ϕ decreases, so does $\sin\phi$, which means ρ increases to compensate.

ρ grows unbounded as ϕ gets smaller & closer to 0.

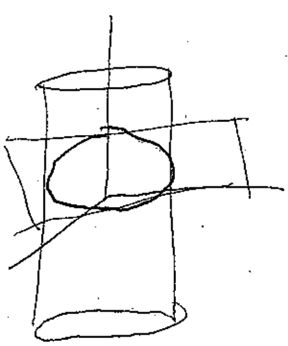
So too, then, does the cylinder.

3. Investigate the curve $\mathbf{r}(t) = \langle \cos(t), \sin(t), \cos(t) + \sin(t) \rangle$. Discuss in your investigation any algebraic relationships between the components and include a rough sketch of the curve.

$$\langle \underset{x}{\cos(t)}, \underset{y}{\sin(t)}, \underset{z}{\cos(t) + \sin(t)} \rangle$$

$x^2 + y^2 = 1$ ← cylinder

$z = y + x$ ← plane



elliptical shape

4. The trajectory of an object is given by the vector valued function

$$\mathbf{r}(t) = \langle 3t^2, t^3, e^t \rangle$$

(a) Find the unit tangent vector to the curve when $t = 2$

$\vec{r}(t) = \langle 3t^2, t^3, e^t \rangle$

$\vec{r}'(t) = \langle 6t, 3t^2, e^t \rangle$

$\vec{r}'(2) = \langle 12, 12, e^2 \rangle$

$|\vec{r}'(2)| = \sqrt{144+144+e^4}$

So unit tangent $= \left\langle \frac{12}{\sqrt{288+e^4}}, \frac{12}{\sqrt{288+e^4}}, \frac{e^2}{\sqrt{288+e^4}} \right\rangle$

(b) Is the object traveling along a smooth curve? Explain.

yes. In general

$|\vec{r}'(t)| = \sqrt{36t^2+9t^4+e^{2t}}$ which is never 0.

5. Determine the equation of the tangent line to

$$\mathbf{r}(t) = \langle e^{t^2}, \sqrt{1+t^2}, t \rangle$$

when $t = 1$.

$\vec{r}(t) = \langle e^{t^2}, \sqrt{1+t^2}, t \rangle$ $\quad$ $r'(t) = \langle 2te^{t^2}, \frac{2t}{\sqrt{1+t^2}}, 1 \rangle$

$\vec{r}(1) = \langle e, \sqrt{2}, 1 \rangle$ $\quad$ $r'(1) = \langle 2e, \frac{2}{\sqrt{2}}, 1 \rangle$

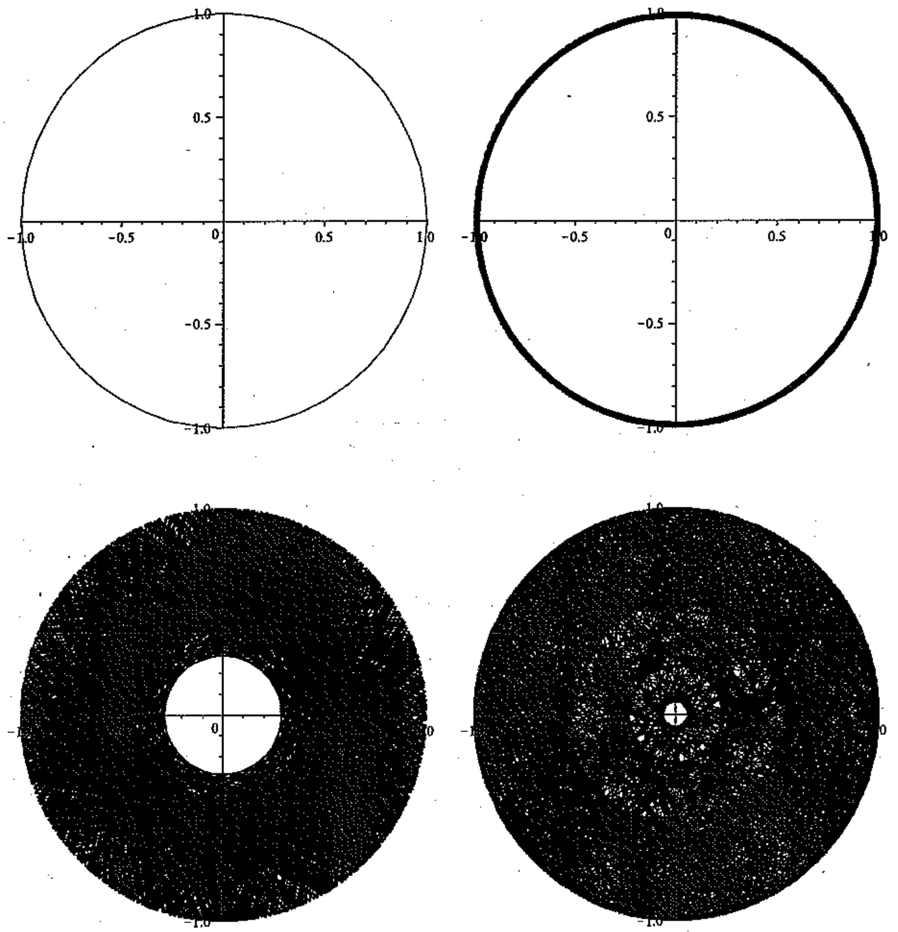
Line

$x = e + 2et$

$y = \sqrt{2} + \frac{2}{\sqrt{2}}t$

$z = 1 + t$

6. (Extra Credit) Below are 4 different computer plots for the parametric equation $\mathbf{r}(t) = \langle \cos(t), \sin(t) \rangle$ for values of t ranging from 0 to 2π, 200π, 2000π and 20000π respectively. Explain why we might get different pictures for each of the graphs.



As the range increases, the computer takes points further and further apart & connects them to draw the graphs, leading to increased imperfections in the drawings.