

Key

Math 225: Quiz the Fourth

This quiz is closed book and closed notes. You may use your calculator for the purposes of arithmetic and computation only. When asked for specific values, however, you must show the relevant algebra. You have the remainder of period.

1. For each of the surfaces below, determine what type of surface it is, what the allowable values are for the requested variables, and what the axis of symmetry is (if it has one).

(a) $x^2 + y^2 + 9z^2 = 81$ (x, y, z values)

ellipsoid

$$-9 \leq x \leq 9$$

$$-9 \leq y \leq 9$$

$$-3 \leq z \leq 3$$

(b) $x^2 + y^2 - 9z^2 = 81$ (z values only)

$$x^2 + y^2 = 81 + 9z^2$$

hyperboloid of one sheet
no restrictions on z

(c) $x^2 + y^2 + 81 = 9z^2$ (z values only)

hyperboloid of 2 sheets

$$9z^2 \geq 81, \quad z \geq 3 \quad \text{or} \quad z \leq -3$$

(d) $x^2 + y^2 + 81 = 9z$ (z values only)

paraboloid (shifted up 81 units)

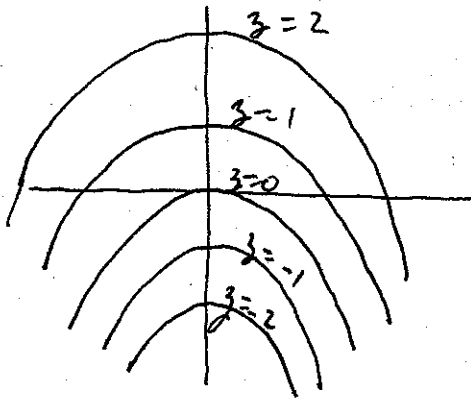
$$z \geq 9$$

2. Consider the function $f(x, y) = x^2 + y$

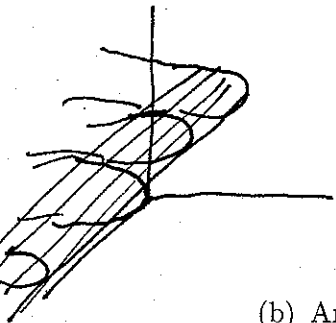
(a) Sketch at least 5 level curves for this function.

$$x^2 + y = k \rightarrow y = k - x^2$$

Parabolas, opening downward



Surface (for interest only)



(b) Are the curves evenly spaced or getting closer and closer together? What does this say about the 'steepness' of the graph of f ?

Curves are evenly spaced, graph has un. form steepness in y

Curves getting closer together in $x \rightarrow$ increasing steepness in x .

3. Find the following limits if they exist.

(a)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + 3y^2}{x^2 + y^2}$$

If $x=0$, then we have $\lim_{y \rightarrow 0} \frac{3y^2}{y^2} = 3$

But if $y=0$, then we have $\lim_{x \rightarrow 0} \frac{x^2}{x^2} = 1$

} so limit DNE.

(b)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 9} - 3}$$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 9} - 3} \cdot \frac{\sqrt{x^2 + y^2 + 9} + 3}{\sqrt{x^2 + y^2 + 9} + 3} \rightarrow \lim_{(x,y) \rightarrow (0,0)} \frac{(x^2 + y^2) \sqrt{x^2 + y^2 + 9} + 3}{(x^2 + y^2 + 9) - 9}$$

$$= \lim_{(x,y) \rightarrow (0,0)} \frac{\sqrt{x^2 + y^2 + 9} + 3}{1} = \boxed{6}$$

4. Consider the function $f(x, y) = 3x^2 - y^2 + 2xy$. Find the tangent plane at the point $(1, 2)$ and use it to approximate $f(1.02, 1.97)$.

$$f(1, 2) = 3 - 4 + 4 = 3$$

$$f_x = 6x + 2y \rightarrow f_x(1, 2) = 6 + 4 = 10$$

$$f_y = (-2y + 2x) \rightarrow f_y(1, 2) = 4 - 2 = 2$$

Tangent plane

$$z = 3 + 10(x - 1) + 2(y - 2)$$

Approximate $f(1.02, 1.97)$

$$z \approx 3 + 10(1.02 - 1) + 2(1.97 - 2)$$

$$= 3 + 10(.02) + 2(-.03)$$

$$= 3 + .2 + .06 = \del{3.19} \\ 3.26$$

5. Extra Credit:

- (a) What is your birthday? (Month and Day only. No years. Please.)

Nov. 19

- (b) Of the 48 of us in the two classes, what is the approximate probability that at least 2 of us have the same birthday?

i. 12 %

ii. 24 %

iii. 48 %

iv. 96 %