Key

Math 225: Quiz the Fifth

This quiz is closed book and closed notes. You may use your calculator for the purposes of arithmetic and computation only. When asked for specific values, however, you must show the relevant algebra. You have the remainder of period.

1. Fill in the blank

   (a) Suppose that f is a function of x and y and each of x and y are functions of s and t. Then
   $$\frac{\partial f}{\partial t} = \underline{\frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}}.$$

   (b) Suppose that $F(x, y, z)$ is a continuous and differentiable function of 3 variables. If $F(x, y, z) = 0$, then we can solve for x and as a function of y and z provided $\underline{F_x \neq 0}$.

   (implicit fcn Thm)

   (c) If $f(x, y)$ is a function of two variables, then $\nabla f = \underline{\langle f_x , f_y \rangle}$

   (d) The gradient vector is <u>perpendicular</u> to the level curve at a point.

2. Let $f(x,y) = \ln(x^2+y^2+1)$. Convert this equation to polar coordinates, compute $\frac{\partial f}{\partial r}$, and verify your answer with the chain rule.

Convert : $x = r\cos\theta$ $\quad y = r\sin\theta$

$$f(r,\theta) = \ln(r^2+1)$$

$$\frac{\partial f}{\partial r} = \frac{1}{r^2+1}(2r)$$

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r}$$

$$= \frac{2x}{x^2+y^2+1}\cdot\cos\theta + \frac{2y}{x^2+y^2+1}\sin\theta$$

$$= \frac{2r\cos^2\theta + 2r\sin^2\theta}{r^2+1} = \frac{2r}{r^2+1}\cancelto{1}{(\cos^2\theta+\sin^2\theta)} = \frac{2r}{r^2+1}, \checkmark, !!, \text{etc.}$$

3. Find the derivative of $f(x,y) = x^4 - 2y^3 + 3x$ at the point (3,-1) in the direction of $\langle 6, 8\rangle$.

$$D_{\vec{u}} f = \vec{\nabla} f \cdot \vec{u}$$

$$\vec{\nabla} f = \langle 4x^3+3, -6y^2\rangle$$

$$\vec{\nabla} f(3,-1) = \langle 111, -6\rangle$$

$$\vec{u} = = \frac{\langle 6,8\rangle}{|\langle 6,8\rangle|} = \left\langle \frac{3}{6}, \frac{4}{5}\right\rangle$$

$$\vec{\nabla} f\cdot\vec{u} = \langle 111, -6\rangle\cdot\left\langle\frac{3}{5},\frac{4}{5}\right\rangle$$

$$= \frac{333}{5} - \frac{24}{5} = \frac{309}{5}$$

4. (a) Suppose that $x^2y + xy + xz + yz = 16$. Find a formula for $\frac{\partial z}{\partial x}$.

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{-(2xy+y+z)}{x+y}$$

(b) Find the equation of the tangent plane to $x^2y + xy + xz + yz = 16$ at the point $(2, 2, 1)$

$$\vec{n} = \vec{\nabla} F = \langle F_x, F_y, F_z \rangle$$

$$= \langle 2xy + y + z, x^2 + x + z, x + y \rangle$$

$$\vec{\nabla} F(2,2,1) = \langle 12, 7, 4 \rangle$$

Plane: $12(x-2) + 7(y-2) + 4(z-1) = 0$

5. We showed in class that $D_{\mathbf{u}}f$ is a maximum when $\mathbf{u}$ is in the direction of ∇f. What is the maximum value that $D_{\mathbf{u}}f$ can attain?

$D_{\vec{u}} f$ is a max when $\vec{u} = \frac{\vec{\nabla} f}{|\vec{\nabla} f|}$

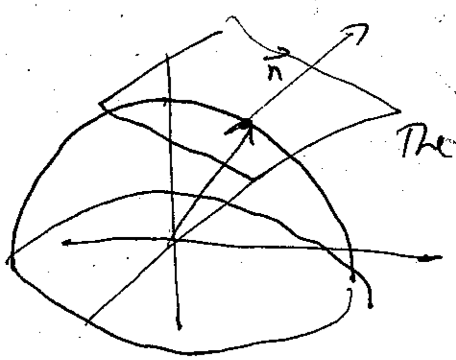
In this case $D_{\vec{u}} f = \vec{\nabla} f \cdot \frac{\vec{\nabla} f}{|\vec{\nabla} f|}$

$$= \frac{(\vec{\nabla} f \cdot \vec{\nabla} f)}{|\vec{\nabla} f|} = \frac{|\vec{\nabla} f|^2}{|\vec{\nabla} f|} = |\vec{\nabla} f| .$$

6. What is the vector normal to the tangent plane of the surface $x^2 + y^2 + z^2 = 1$ at the point (a, b, c)? Interpret your answer geometrically.

$\vec{n} = \vec{\nabla} F = \langle 2x, 2y, 2z \rangle$

$\vec{\nabla} F(a,b,c) = \langle 2a, 2b, 2c \rangle$ which is parallel to the radius



The radius is perpendicular to the tangent plane

7. (Extra Credit) You may take either one half-point for yourself, or zero points for yourself and give one-twentieth of a point to everyone else in the class. Your extra credit score will be the total of points taken for yourself and given to you by others.

Since I'm not being graded, I'll kick in 1/20 of a point for everyone