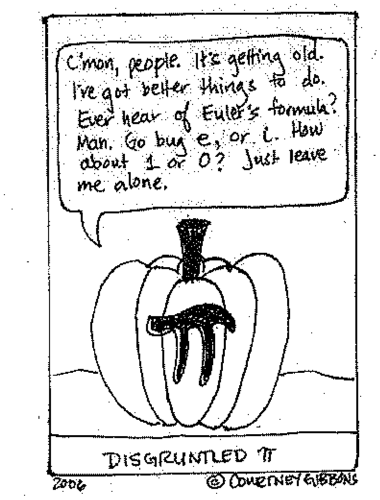
KEY

Math 225: Quiz the Sixth
October 31, 2008

You know the drill by now. No books, no notes, no colleagues, and no answers without justification.
READ ALL QUESTIONS CAREFULLY



1. (a) Suppose that a continuous, differentiable function $f(x, y)$ has a local maximum at the point (a, b). What can we conclude about the partial derivatives (both first and second order) of f at (a, b)?

Both f_x and f_y are zero

$D = f_{xx} f_{yy} - (f_{xy})^2 \geq 0$ and $f_{xx} < 0$

(b) Suppose that we want to maximize a function $f(x, y)$ with regard to the constraint $g(x, y) = k$. Then $\nabla f =$ $\lambda \vec{\nabla} g$.

(c) Clairaut's Theorem says that for a continuous, twice-differentiable function with continuous partial derivatives, that $f_{xy} = f_{yx}$.

2. Find and classify the critical point of $f(x, y) = x^3y + 12x^2 - 8y$.

$f_x = 3x^2y + 24x = 0$

$f_y = x^3 - 8 = 0$

$x = 2 \rightarrow 12y + 48 = 0 \quad y = -4$

c.p. $= (2, -4)$

$D = f_{xx} f_{yy} - (f_{xy})^2$

$(12xy + 24)(0) - (3x^2)^2$

$= -144 < 0$, so $(2, -4)$ is a saddle point.

3. Using Lagrange Multipliers, find the maximum value of $f(x, y) = 2x^2 + 3y^2 - 4x - 5$ subject to $x^2 + y^2 = 16$.

$f = 2x^2 + 3y^2 - 4x - 5 \qquad g = x^2 + y^2$

$\vec{\nabla} f = \lambda \vec{\nabla} g$

$4x - 4 = 2\lambda x$

$6y = 2\lambda y \qquad y = 0 \text{ or } \lambda = 3$

$x^2 + y^2 = 16$

$y = 0 \Rightarrow x = \pm 4$

$\lambda = 3 \Rightarrow 4x - 4 = 6x$

$x = -2$

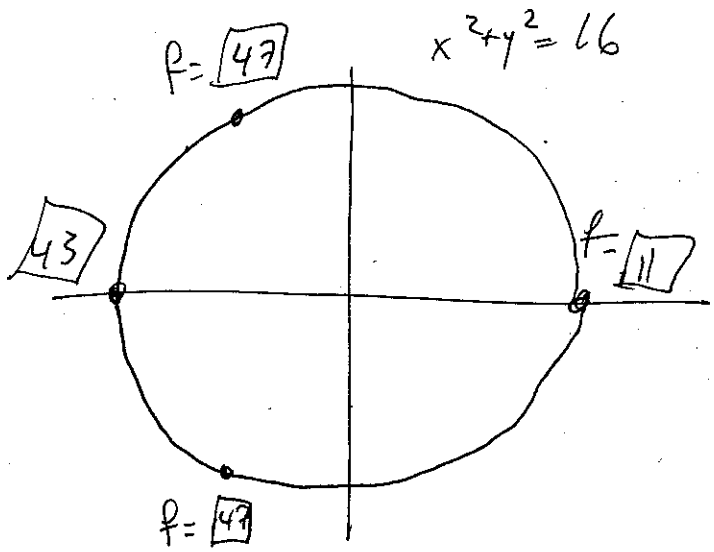
$y = \pm\sqrt{12}$

Test $f(4,0) = 32 + 0 - 16 - 5 = 11$

$\cancel{f(-4,0) = 32 + 0 + 16 - 5 = 43} \leftarrow \cancel{\text{max}}$

$f(-2, \sqrt{12}) = 8 + 36 + 8 - 5 = \cancel{23}\ 47$

$f(-2, -\sqrt{12}) = 8 + 36 + 8 - 5 = \cancel{23}\ 47$ max



4. Using the method of your choice, find the point on the plane $x + 2y + 3z = 10$ that is closest to the origin.

g

$f = x^2 + y^2 + z^2$

I choose Lagrange...

$\vec{\nabla} F = \lambda \vec{\nabla} g$

$2x = \lambda$

$2y = 2\lambda$

$2z = 3\lambda$

$x + 2y + 3z = 10$

$\frac{\lambda}{2} + 2\lambda + \frac{9\lambda}{2} = 10$

$\frac{1\cancel{4}\lambda}{2} = 10$

$\lambda = \frac{20}{14}$

$\boxed{x = \frac{20}{28} \quad y = \frac{20}{14} \quad z = \frac{60}{28}}$