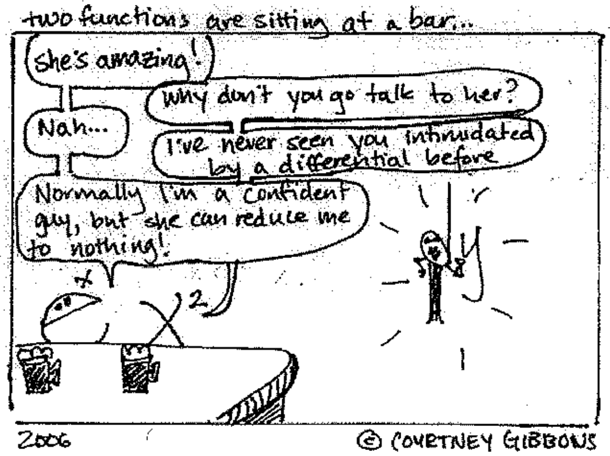
KEY

Math 225: Quiz the Seventh
November 7, 2008

You know the drill by now. No books, no notes, no colleagues, and no answers without justification.
READ ALL QUESTIONS CAREFULLY



dy walks into a bar...

1. (a) Geometrically, what does $\int\int_R f(x,y)dA$ calculate?

The volume under $z = f(x,y)$ and above R.

(b) Geometrically, what does $\int\int_R 1dA$ calculate?

The area of R (World's Most Boring Example)

(c) $\int_a^b \int_c^d f(x,y)dy\,dx = \int_c^d \int_a^b f(x,y)dx\,dy$ according to whose theorem?

Fubini's Theorem

2. Find the volume beneath the plane $3x + 2y + z = 8$ and above the rectangle $[0, 1] \times [0, 2]$

$$\int_0^1 \int_0^2 8 - 3x - 2y \, dy \, dx$$

$$= \int_0^1 8y - 3xy - y^2 \Big]_0^2$$

$$= \int_0^1 16 - 6x - 4 \, dx = \int_0^1 12 - 6x \, dx$$

$$= 12x - 3x^2 \Big]_0^1 = 9$$

3. Find

$$\int_{-1}^{1} \int_{-1}^{1} x^2 y^3 \, dx \, dy$$

and explain why your answer make sense.

$$\int_{-1}^{1} \int_{-1}^{1} x^2 y^3 \, dx \, dy = \int_{-1}^{1} \frac{x^3}{3} y^3 \Big]_{-1}^{1} dy$$

$$= \int_{-1}^{1} \frac{2}{3} y^3 \, dy$$

$$= \left[\frac{1}{6} y^4 \right]_{-1}^{1} = \frac{1}{6} - \frac{1}{6} = 0$$

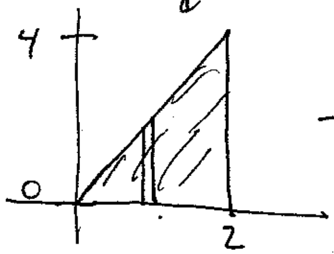
Since y^3 is odd, we have the same volume above & below xy plane.

4. Solve the following integrals by first reversing the order of integration.

(a)

$$\int_0^4 \int_{\frac{y}{2}}^2 e^{x^2}\, dx\, dy$$

region



$$\to \int_0^2 \int_0^{2x} e^{x^2}\, dy\, dx$$

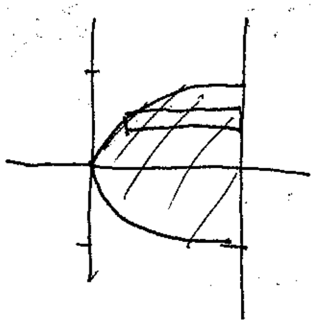
$$= \int_0^2 y e^{x^2} \Big]_0^{2x} dx$$

$$= \int_0^2 2x e^{x^2}\, dx =$$

$$e^{x^2}\Big]_0^2 = \underline{e^4 - 1}$$

(b)

$$\int_0^9 \int_{-\sqrt{x}}^{\sqrt{x}} x + y\, dy\, dx$$



$$\int_{-3}^3 \int_{y^2}^9 x + y\, dx\, dy$$

$$= \int_{-3}^3 \frac{x^2}{2} + xy \Big]_{y^2}^9 dy$$

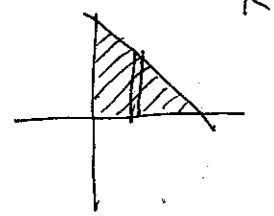
$$= \int_{-3}^3 \frac{81}{2} + 9y - \frac{y^4}{2} - y^3\, dy$$

$$= \Big[\frac{81}{2}y + \frac{9}{2}y^2 - \frac{y^5}{10} - \frac{y^4}{4} \Big]_{-3}^3$$

$$= \left[\frac{243}{2} + \cancel{\frac{81}{2}} - \frac{243}{10} - \cancel{\frac{81}{4}}\right] - \left[-\frac{243}{2} + \cancel{\frac{81}{2}} + \frac{243}{10} - \cancel{\frac{81}{4}}\right]$$

$$= 243 - \frac{243}{5} = \frac{972}{5}$$

(c) Find the volume of the tetrahedron bound by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$.



$$\int_0^1 \int_0^{1-x} 1 - x - y \; dy\, dx$$

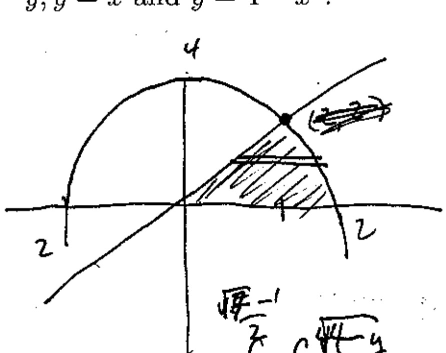
$$\int_0^1 y - xy - \frac{y^2}{2} \; dx$$

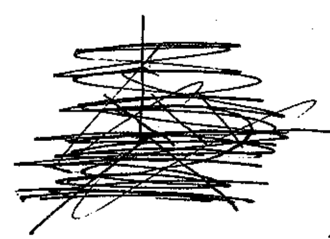
$$= \int_0^1 (1-x) - x(1-x) - \frac{(1-x)^2}{2} \, dx$$

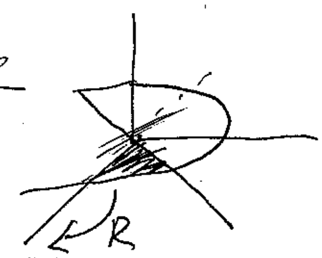
$$= \int_0^1 1 - 2x + x^2 - \frac{(1-2x+x^2)}{2} \, dx = \frac{1}{2}\int_0^1 (1-x)^2$$

$$= -\frac{1}{6}(1-x)^3 \Big|_0^1 = \boxed{\frac{1}{6}}$$

(d) Set up, but DO NOT EVALUATE an integral to find the volume bound by $z = 0, z = y, y = x$ and $y = 4 - x^2$.







$$\int_0^{\frac{\sqrt{17}-1}{2}} \int_y^{\sqrt{4-y}} y \; dx\, dy$$

$$4 - x^2 = x$$

$$x^2 + x - 4 = 0$$

$$\frac{-1 \pm \sqrt{17}}{2}$$

(Extra Credit) Pretend to flip a coin 5 times and write down what you get (as a string of heads and tails). (one-half point)

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