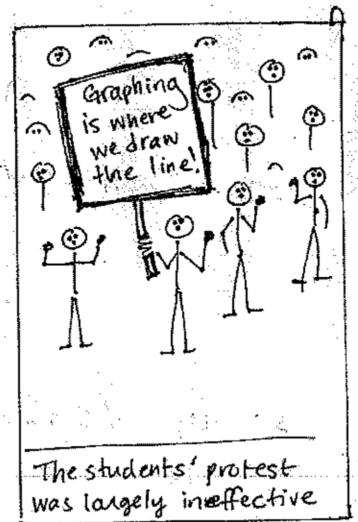
Key

Math 225: Quiz the Last

Dec 10, 2008

You know the drill by now. No book, no notes, no colleagues, and no answers without justification.



The students' protest was largely ineffective

1. Short Answer

   (a) What conditions on partial derivatives ensures that $\langle P, Q\rangle$ is a conservative vector field?

   $$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$$

   (b) The Fundamental Theorem of Line Integrals states that $\int_C \nabla(f) \cdot d\mathbf{r} =$

   $f(b) - f(a)$, where by "b" and "a", we mean the endpoints of the curve

   (c) Green's Theorem states that $\int\int_R \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA = \oint_{\partial R} P\,dx + Q\,dy$

2. Determine whether the following vector fields are conservative. For those that are, find $f(x, y)$ such that $\mathbf{F} = \nabla f$.

(a) $\mathbf{F} = \langle 2, 3 \rangle$

(b) $\mathbf{F} = \langle \cos(y), \sin(x) \rangle$

(c) $\mathbf{F} = \langle e^x + y, e^{2y} + x \rangle$

(d) $\mathbf{F} = \langle x^2 + xy, xy + y^2 \rangle$

a) $\frac{\partial}{\partial x}(3) = \frac{\partial}{\partial y}(2) = 0$, so it's conservative.

$f = 2x + 3y$

b) $\frac{\partial}{\partial x}(\sin(x)) = \cos x$

$\frac{\partial}{\partial y}(\cos(y)) = -\sin y$ so no, not conservative

c) $\frac{\partial}{\partial x}(e^{2y} + x) = 1$, so it's conservative

$\frac{\partial}{\partial y}(e^x + y) = 1$

$f = e^x + xy + \frac{1}{2}e^{2y}$

d) $\frac{\partial}{\partial x}(xy + y^2) = y$ so no, not conservative

$\frac{\partial}{\partial y}(x^2 + xy) = x$

3. Evaluate $\int_C 2xyz\; dx + x^2z\; dy + x^2y\; dz$ where C is the line segment from $(1,1,1)$ to $(1,2,4)$.

$$\langle 2xyz, x^2z, x^2y \rangle = \vec{\nabla}(x^2yz)$$

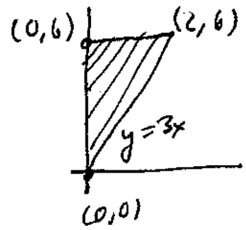
$$\text{so } \int_C 2xyz\,dx + x^2z\,dy + x^2y\,dz = x^2yz\Big|_{(1,1,1)}^{(1,2,4)} = 8-1 = 7$$

4. Evaluate $\int_C 2xyz\; dx + x^2z\; dy + x^2y\; dz$ where C is the curve given by $\mathbf{r}(t) = \langle 1, t, t^2 \rangle$ from $(1,1,1)$ to $(1,2,4)$.

Since $\int_C \vec{F}\cdot d\vec{r}$ is independent of path, here, also

$$\int_C \vec{F}\cdot d\vec{r} = 7$$

5. Find $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $F = \langle y^2 + e^x, x + e^{2y} \rangle$ and C is the triangle from $(0,0)$ to $(2,6)$ to $(0,6)$ to $(0,0)$.



$$\oint_C \vec{F} \cdot d\vec{r} = \cancel{\iint_R y^2 + e^x}$$

$$= \iint_R \frac{\partial}{\partial x}(x + e^2 y) - \frac{\partial}{\partial y}(y^2 + e^x)\, dA$$

$$= \int_0^6 \int_0^{\frac{1}{3}y} 1 - 2y \, dx\, dy$$

$$= \int_0^6 \frac{1}{3}y - \frac{2}{3}y^2 \, dy$$

$$= \frac{y^2}{6} - \frac{2y^3}{9} \Big|_0^6 = \frac{36}{6} - \frac{2 \cdot 216}{9} = 6 - 48 = -42$$

6. (Extra Credit) What is the sum of the first *200* numbers?

$1 + 200 = 201$

$2 + 199 = 201$

... always 201 ...

$3 + 198 = 201$

Answer: $100 \times 201 = \underline{20100}$