Math 225: Practice Exam the First

Fall 2009

These questions come, for the most part, directly from old exams. They are representative of the types of questions that you'll be likely to see on Wednesday's exam.

1. (a) Give the equation of the (circular) cylinder of radius 6 centered around the z-axis.
   (b) Give the equation of the (circular) cylinder of radius 6 centered around the y-axis.
   (c) Find, in parametric form, the equation of the curve of intersection of the cylinder in part (b) with the plane $y + 4z = 3$, and describe the curve.

2. Let ℓ_1 be the line through the two points (-3, 1, 0) and (1,1,2), and ℓ_2 be the line through the points (6, 2, 6) and (3, -1, 0).

   (a) Find the point of intersection of ℓ_1 and ℓ_2.
   (b) Find the plane that contains both lines.

3. (a) Find the equation of the plane that contains the points $(4, 1, 1), (5, 3, 3)$ and $(7, 1, 2)$.
   (b) Is the triangle determined by these three points acute, right, or obtuse? Explain.

4. True or False, and why?

   (a) Two planes parallel to each of two skew lines are themselves parallel.
   (b) Two planes parallel to each of two parallel lines are themselves parallel.
   (c) Two planes can intersect in exactly one point.
   (d) Three planes can intersect in exactly one point
   (e) Four planes can intersect in exactly one point.

5. Prove the parallelogram law, which states that for vectors x and y

$$|\mathbf{x} + \mathbf{y}|^2 + |\mathbf{x} - \mathbf{y}|^2 = 2|\mathbf{x}|^2 + 2|\mathbf{y}|^2$$

6. Let $\mathbf{x}$ and $\mathbf{y}$ be unit vectors. What are the minimum and the maximum magnitude of $\mathbf{x} \times \mathbf{y}$, and what is the geometric relationship between $\mathbf{x}$ and $\mathbf{y}$ when these are achieved? Why?

7. Suppose that a particle is moving with acceleration

$$\mathbf{a}(t) = \langle 6t, \cos(t), e^t \rangle$$

   and that the object starts with initial velocity vector $\langle 2, 1, 2 \rangle$ and initial position vector $\langle 0, 1, 3 \rangle$. Find the position of the object when $t = 1$.

8. The function $\mathbf{r}(t) = \langle t\cos(t), t\sin(t), t \rangle$ has a 'conical helix' as its graph.

   (a) Find the equation of the tangent line to this helix when $t = \frac{\pi}{2}$

(b) Set up, but don't compute, an integral which calculates the arc length of this curve on its first rotation up from the origin.

(c) Describe what happens to the curvature as t increases. DO NOT try to calculate an explicit formula for the curvature.

9. (a) Find the equation of the tangent line to the curve $\mathbf{r}(t) = \langle 4 - t, 3t - t^2, t \rangle$ at the point when $t = 0$.

   (b) Using your work in part (a), find $\mathbf{T}(0)$. (Do NOT try to calculate a generic formula for $\mathbf{T}(t)$).

   (c) We can show that $\mathbf{T}'(0) = \langle -3, -2, 3 \rangle$. Find $\mathbf{N}(0)$, $\mathbf{B}(0)$.

10. (a) Give the formula for the binormal vector to a curve, $\mathbf{B}(t)$.

    (b) Differentiate your formula to find a formula for $\mathbf{B}'(t)$.

    (c) Prove that $\mathbf{B}'(t)$ is perpendicular to $\mathbf{T}(t)$.

11. For each surface, determine what type of surface it is, and what the allowable values are fo the requested variables.

    (a) $x^2 + 72y^2 + 18z^2 = 288$ (x, y, z values)

    (b) $x^2 + 72y^2 - 18z^2 = 288$ (z values only)

    (c) $x^2 + 72y^2 + 288 = 18z^2$ (z values only)

    (d) $x^2 + y^2 + 288 = 72z$ (z values only)