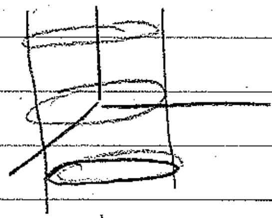
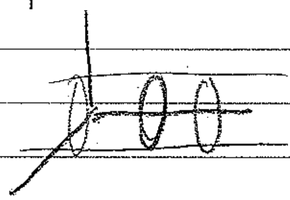
1a) radius 6 centered z axis



$x^2+y^2=36$

b.



$x^2+z^2=36$

curve of intersection

c. $x^2+z^2=36$ $\quad$ $y+4z=3$

$x=6\cos\theta$
$y=3-24\sin\theta$
$z=6\sin\theta$

or

$x=t$
$z=3\mp4\sqrt{36-t^2}$
$z=\pm\sqrt{36-t^2}$

forms an ellipse

2. l_1 $(-3,1,0)$ and $(1,1,2)$ $\quad$ $\langle 4,0,2\rangle$ $\quad$ $x=1+4t$, $y=1$, $z=2+2t$

l_2 $(6,2,6)$ and $(3,-1,0)$

$\langle -3,-3,-6\rangle$

$\langle 3,3,6\rangle$

$x=3+3s=1+4t$ $\quad$ $1+4t=5$ $\quad$ $t=1$ $\quad$ $(5,1,4)$

$y=-1+3s=1$ $\quad$ $s=2/3$ $\quad$ pt. of intersection

$z=6s=2+2t$

b) $\vec{n}=\langle 4,0,2\rangle\times\langle 3,3,6\rangle=\langle -6,-18,12\rangle$

$-6(x-5)-18(y-1)+12(z-4)=0$

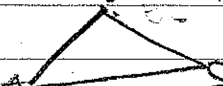
3a $\overset{A}{(4,1,1)}$ $\overset{B}{(5,3,3)}$ $\overset{C}{(7,1,2)}$

$\overrightarrow{AB}(1,2,2)$ $\quad$ $\overrightarrow{AC}(3,0,1)$

$\times(3,0,1)$

$\langle 2,5,-6\rangle$

$2(x-4)+5(y-1)-6(z-1)=0$



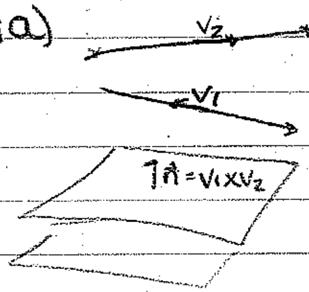
B $\overrightarrow{BA}\cdot\overrightarrow{BC}$ (+)

$\overrightarrow{AB}=\langle 1,2,2\rangle$
$\overrightarrow{AC}=\langle 3,0,1\rangle$
$\overrightarrow{BC}=\langle 2,-2,-1\rangle$

$\overrightarrow{AB}\cdot\overrightarrow{AC}$ (+)

$\overrightarrow{CA}\cdot\overrightarrow{CB}$ (+)

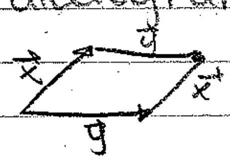
$\overrightarrow{BA}(-1,-2,-2)$
$\overrightarrow{CA}(-3,0,-1)$
$\overrightarrow{CB}(-2,2,1)$

4a) 

True

b) false $v_1 = v_2$ so can't compute $\vec{n}$
c) false intersect at a line
d) true coordinate planes
e) true coordinate planes plus a plane through origin

5. $|x+y|^2 + |x-y|^2 = 2|x|^2 + 2|y|^2$
$(x+y)\cdot(x+y)$ (
$x \cdot x + \cancel{2(x \cdot y)} + y \cdot y + x \cdot x - \cancel{2(x \cdot y)} + y \cdot y$
$2(x \cdot x) + 2(y \cdot y)$
$2|\vec{x}|^2 + 2|\vec{y}|^2$

6. x and y are unit vectors
$|x \times y| = |x||y|\sin\theta$ also area of parallelogram
max magnitude $= 1$ when $\theta = \pi/2$
min magnitude $= 0$ when $\theta = 0$


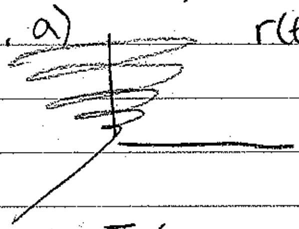
7. $a(t) = \langle 6t, \cos(t), e^t \rangle$
initial velocity $\langle 2,1,2 \rangle$ initial position $\langle 0,1,3 \rangle$

$\langle 2,1,2 \rangle + \int_0^t \langle 6u, \cos(u), e^u \rangle du$

$\langle 2,1,2 \rangle + \langle 3u^2, \sin(u), e^u \rangle |_0^t$
$v(t) = \langle 3t^2 + 2, \sin(t) + 1, e^t + 1 \rangle$
$s(t) = \langle 0,1,3 \rangle + \int_0^t \langle 3u^2 + 2, \sin u + 1, e^u + 1 \rangle du$
$= \langle 0,1,3 \rangle + \langle u^3 + 2u, -\cos u + u, e^u + u \rangle |_0^t$
$= \langle 0,1,3 \rangle + \langle t^3 + 2t, -\cos t + t + 1, e^t + t - 1 \rangle$
$= \langle t^3 + 2t, -\cos t + t + 2, e^t + t + 2 \rangle$

$s(1) = \langle 3, 3-\cos(1), e+3 \rangle$

8. a) $r(t) \langle t\cos t, t\sin t, t \rangle$



$x^2+y^2 = t^2 = z^2$

$x^2+y^2 = z^2$

$r'(t) = \langle \cos t - t\sin t, \sin t + t\cos t, 1 \rangle$

$t = \pi/2$ $\quad r'(\pi/2) = \langle -\pi/2, 1, 1 \rangle$

$r(\pi/2) = \langle 0, \pi/2, \pi/2 \rangle$

line $=$ $x = 0 - \pi/2 t$

$y = \pi/2 + t$

$z = \pi/2 + t$

b)

$\int_0^{2\pi} |r'(t)| dt \qquad \int_0^{2\pi} \sqrt{(\cos t - t\sin t)^2 + (\sin t + t\cos t)^2 + 1}$

$\int_0^{2\pi} \sqrt{2+t^2}\, dt$

c) curvature get's less and less curvy as t increases

9. $r(t) = \langle 4-t, 3t-t^2, t \rangle \quad t = 0$

$r(0) = \langle 4, 0, 0 \rangle$

$r'(t) = \langle -1, 3-2t, 1 \rangle$

$r'(0) = \langle -1, 3, 1 \rangle$

line $\langle 4,0,0 \rangle + t\langle -1,3,1 \rangle$

b) $\vec{T}(0) = \dfrac{r'(0)}{|r'(0)|} \quad \dfrac{\langle -1,3,1 \rangle}{\sqrt{11}} = \left\langle \dfrac{-1}{\sqrt{11}}, \dfrac{3}{\sqrt{11}}, \dfrac{1}{\sqrt{11}} \right\rangle$

$N(0) = \dfrac{T'(0)}{|T'(0)|} = \dfrac{\langle -3,-2,3 \rangle}{\sqrt{22}} = \left\langle \dfrac{-3}{\sqrt{22}}, \dfrac{-2}{\sqrt{22}}, \dfrac{3}{\sqrt{22}} \right\rangle$

$B(0) = T \times N = \left\langle \dfrac{11}{11\sqrt{2}}, 0, \dfrac{11}{11\sqrt{2}} \right\rangle = \left\langle \dfrac{1}{\sqrt{2}}, 0, \dfrac{1}{\sqrt{2}} \right\rangle$

osculating plane $\dfrac{1}{\sqrt{2}}(x-4) + \dfrac{1}{\sqrt{2}}(z) = 0$

$\vec{n} = \vec{B}$ point $r(0)$

10\. a) $B(t) = T(t) \times N(t)$

b) $(T(t) \times N(t))'$

$T'(t) \times N(t) + T(t) \times N'(t)$

$\cancel{T'(t) \times \frac{T'(t)}{|T'(t)|}} + T(t) \times N'(t)$

parallel vectors crossed = 0

$B'(t) = T(t) \times N'(t)$

$B'(t) \perp T(t)$

11a) $x^2 + 72y^2 + 18z^2 = 288$ ellipsoid

$\frac{x^2}{288} + \frac{y^2}{4} + \frac{z^2}{16} = 1$ $\quad -4 \le z \le 4$

$-2 \le y \le 2$

$-\sqrt{288} \le x \le \sqrt{288}$

b) $x^2 + 72y^2 = 288 + 18z^2$

z = all real

side with one term hyperboloid of one sheet

c) $x^2 + 72y^2 + 288 = 18z^2$

side with two terms hyperboloid of two sheets

$z \ge 4 \quad z \le -4$

d) $x^2 + y^2 + 288 = 72z$ elliptical paraboloid

$\frac{x^2}{72} + \frac{y^2}{72} + 4 = z \qquad z > 4$