KEY

Math 225: PreQuiz

These problems are representative of the type that you have encountered in your mathematical past, and give some of the concepts that you'll be responsible for as we move through the new material. This will not be collected, though I will provide a solutions key online.

1. Give the domain and range for the function $f(x) = \ln(1 - x^2)$.

$\ln t$ is defined for $t > 0$

$\ln(1-x^2)$ is defined for $1 - x^2 > 0$

$x^2 < 1$

Domain: $-1 < x < 1$

Range: for $0 < t < 1$, $\ln t < 0$

Range $\ln(1-x^2)$ is $(-\infty, 0]$

2. Find

$$\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 5x + 6}$$

$$\lim_{x \to 2} \frac{x^2-3x+2}{x^2-5x+6} = \lim_{x \to 2} \frac{(x-2)(x-1)}{(x-2)(x-3)} = \lim_{x \to 2} \frac{(x-1)}{(x-3)} = \frac{1}{-1} = -1$$

3. Find the maximum and minimum values of $f(x) = 3x - x^2$ on the interval $[1, 4]$

Crit. points

$f(x) = 3x - x^2$

$f'(x) = 3 - 2x = 0$

$x = 3/2$

To check

crit points → $f(\frac{3}{2}) = \frac{9}{2} - \frac{9}{4} = \frac{9}{4}$

$f(1) = 3 - 1 = 2$

endpoints → $f(4) = 12 - 16 = -4$

max value: $\frac{9}{4}$

min value: -4

4. Let $f(x) = \sqrt[3]{x}$. Find the tangent line to $f(x)$ at $x = 8$ and use it to approximate $\sqrt[3]{8.02}$.

$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

$f(8) = 2$

$f'(x) = \frac{1}{3} x^{-2/3}$

$f'(8) = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$

Tangent line: $y = f(a) + f'(a)(x - a)$

$y = 2 + \frac{1}{12}(x - 8)$

$\sqrt[3]{8.02} \approx 2 + \frac{1}{12}(.02) \approx 2.0016666$

KEY

5. Find

Improper (div. by 0!)

$$\int_0^1 \frac{t\,dt}{\sqrt{1-t^4}} = \lim_{b\to 1^-} \int_0^b \frac{t\,dt}{\sqrt{1-t^4}}$$

let $u = t^2$

$du = 2t\,dt$

$$\lim_{b\to 1^-} \frac{1}{2}\int_0^{b^2} \frac{du}{\sqrt{1-u^2}}$$

$$= \lim_{b\to 1^-} \frac{1}{2}\arcsin u \Big]_0^{b^2} = \lim_{b\to 1^-} \frac{1}{2}\arcsin b^2 - \frac{1}{2}\arcsin 0$$

$$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

6. Find

$$\int \cos^2\theta\, d\theta$$

$$\int \cos^2\theta\, d\theta = \int \frac{1+\cos 2\theta}{2}\, d\theta = \frac{\theta}{2} + \frac{\sin 2\theta}{4} + C$$

7. Find

$$\int_2^4 \ln(3x)\, dx$$

let $y = 3x$

$dy = 3\,dx$

$$\frac{1}{3}\int_6^{12} \ln y\, dy = \frac{1}{3}\left[y\ln y - \int y\cdot\frac{1}{y}\, dy\right]_6^{12}$$

$u = \ln y \quad dv = dy$

$du = \frac{1}{y}dy \quad v = y$

$$= \frac{1}{3}\left[y\ln y - y\right]_6^{12}$$

$$= \frac{1}{3}\left[12\ln 12 - 12 - 6\ln 6 + 6\right]$$

$$= \left[4\ln 12 - 2\ln 6 - 2\right]$$