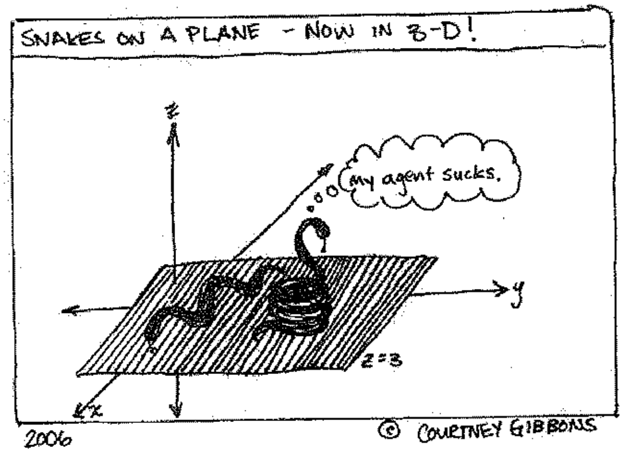
KEY 09.09.09

Math 225: Quiz the First

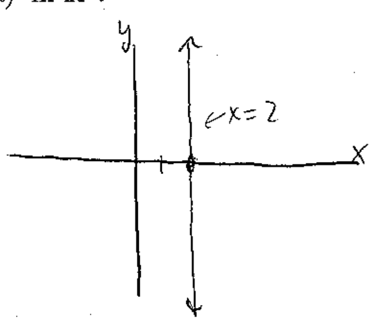
This quiz is closed book and closed notes. You may use your calculator for the purposes of arithmetic operations (including trig). When asked for specific values, however, you must show the relevant algebra. **READ ALL QUESTIONS CAREFULLY!** You have the remainder of the period.



1. Plot and describe the set of points satisfied by $x = 2$

   (a) in $\mathbb{R}^2$.

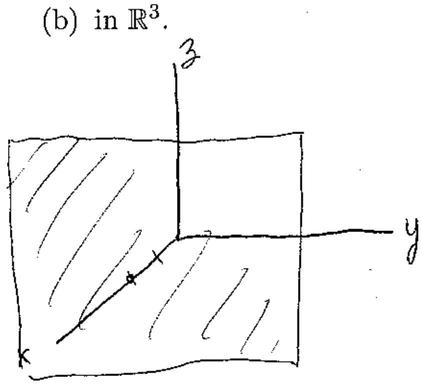
(2)



line parallel to y-axis through (2,0)

   (b) in $\mathbb{R}^3$.

(2)



plane parallel to y-z plane through (2,0,0)

2. Give the equation for the sphere centered at $(4, 2, -1)$ with radius 3. Also, tell whether this sphere intersects each of the coordinate (xy, xz, and yz) ~~axes~~. planes

Eq: $(x-4)^2+(y-2)^2+(z+1)^2=9$

This sphere intersects the xz and xy planes, but not the yz plane (as the x coordinate of the center is too far away)

5

3. Let $A = (2, 1, 3)$, $B = (4, -1, 3)$, and $C = (0, 3, 2)$

(a) Find $\overrightarrow{AB}$, $\overrightarrow{AC}$ and their magnitudes.

$\overrightarrow{AB} = \langle 4-2, -1-1, 3-3 \rangle = \langle 2, -2, 0 \rangle$ $\quad |\overrightarrow{AB}| = \sqrt{8}$

$\overrightarrow{AC} = \langle 0-2, 3-1, 2-3 \rangle = \langle -2, 2, -1 \rangle$ $\quad |\overrightarrow{AC}| = \sqrt{9} = 3$

2

(b) Find $\overrightarrow{AB} \cdot \overrightarrow{AC}$.

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 2\cdot(-2) + (-2)\cdot 2 + 0 = -8$

2

(c) Without calculating the angle, is the angle formed at A acute, right, or obtuse? Explain.

The angle at A is obtuse, as $(\overrightarrow{AB} \cdot \overrightarrow{AC}) < 0$.

2

4. (a) Let $\mathbf{x} = \langle 1, 1 \rangle$ and $\mathbf{y} = \langle 1, -1 \rangle$ and $\mathbf{z} = \langle 3, 5 \rangle$. Find s, t such that $\mathbf{z} = s\mathbf{x} + t\mathbf{y}$.

$$\langle 3,5 \rangle = s\langle 1,1 \rangle + t\langle 1,-1 \rangle$$

$$3 = s + t$$
$$5 = s - t$$
$$8 = 2s \qquad s = 4 \quad t = -1$$

2

(b) Find $\text{proj}_{\mathbf{x}}\mathbf{z}$ and $\text{proj}_{\mathbf{y}}\mathbf{z}$.

$$\text{proj}_{\vec{x}} \vec{z} = \frac{\vec{z} \cdot \vec{x}}{|\vec{x}|^2} (\vec{x}) = \frac{\langle 3,5 \rangle \cdot \langle 1,1 \rangle}{2} \langle 1,1 \rangle = \langle 4,4 \rangle$$

$$\text{proj}_{\vec{y}} \vec{z} = \frac{\vec{z} \cdot \vec{y}}{|\vec{y}|^2} \vec{y} = \frac{\langle 3,5 \rangle \cdot \langle 1,-1 \rangle}{2} \langle 1,-1 \rangle = \langle 1, +1 \rangle$$

3

5. Prove that if $\mathbf{x}+\mathbf{y}$ is orthogonal to $\mathbf{x}-\mathbf{y}$, then $|\mathbf{x}| = |\mathbf{y}|$ (Hint: Calculate an appropriate dot product).

$$\vec{x}+\vec{y} \perp \vec{x}-\vec{y}$$

$$\Rightarrow (\vec{x}+\vec{y})\cdot(\vec{x}-\vec{y}) = 0$$

$$\Rightarrow \vec{x}\circ\vec{x} + \cancel{\vec{y}\cdot\vec{x}} - \cancel{\vec{x}\cdot\vec{y}} - \vec{y}\circ\vec{y} = 0$$

5

$$\vec{x}\circ\vec{x} - \vec{y}\circ\vec{y} = 0 \Rightarrow \vec{x}\circ\vec{x} = \vec{y}\circ\vec{y}$$

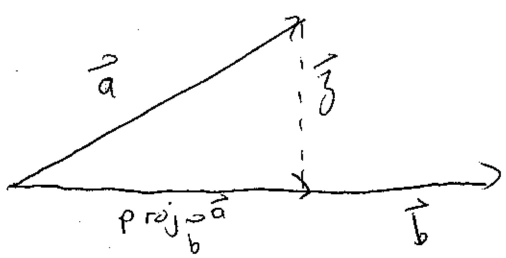
$$|\vec{x}|^2 = |\vec{y}|^2 \Rightarrow |\vec{x}| = |\vec{y}|$$

✓, ☺, etc.

6. (Bonus) Might it ever be the case that $\text{proj}_{\mathbf{b}}\mathbf{a}$ is longer than $\mathbf{a}$? Explain.

No. This will never occur.

up to 2



Note that $\vec{a} = \text{proj}_{\vec{b}}\vec{a} + \vec{z}$

and, since $\text{proj}_{\vec{b}}\vec{a} \perp \vec{z}$, we have

$$|\vec{a}|^2 = |\text{proj}_{\vec{b}}\vec{a}|^2 + |\vec{z}|^2$$

The projection will always be at most as long as the original vector!

or $|\vec{a}|^2 \geq |\text{proj}_{\vec{b}}\vec{a}|^2$

$$|\vec{a}| \geq |\text{proj}_{\vec{b}}\vec{a}|$$