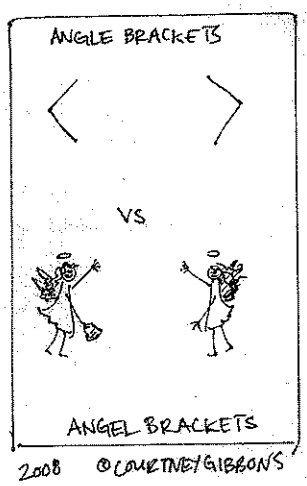
KEY

Math 225: Quiz the Second

This quiz is closed book and closed notes. You may use your calculator for the purposes of arithmetic and for plotting equations, if helpful. When asked for specific values, however, you must show the relevant algebra. PLEASE READ ALL QUESTIONS CAREFULLY. You have the remainder of the period.



1. Let $\mathbf{x}, \mathbf{y}$ and $\mathbf{z}$ be vectors in $\mathbb{R}^3$. For each quantity listed, tell whether it is a vector, a scalar, or if the quantity does not make sense.

(a) $\frac{1}{|\mathbf{x}|}(\mathbf{y} \times \mathbf{z})$ vector

sc vec

(b) $(\mathbf{x} \times \mathbf{y}) + \mathbf{z}$ vector

vec vec

(5) (c) $(\mathbf{x} \cdot \mathbf{y}) \times \mathbf{z}$ nonsense

sc vec

(d) $(\mathbf{x} \cdot \mathbf{z}) + (\mathbf{y} \cdot \mathbf{z})$ scalar

sc sc

(e) $\mathbf{x}^2$ nonsense

2. Suppose that we take the cross product of two vectors, each of length 1? Must the resulting vector be a vector of length 1? Might it? Explain.

$|\vec{u}| = 1$

$|\vec{v}| = 1$

$$|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin\theta$$

$$= 1 \cdot 1 \cdot \sin\theta$$

so $|\vec{u} \times \vec{v}| = 1 \iff \sin\theta = 1 \iff \theta = \frac{\pi}{2}$

so only if the vectors are at right angles.

(4)

3. Do the vectors $\langle 2, 3, 1\rangle$, $\langle 1, -1, 0\rangle$, and $\langle 7, 3, 2\rangle$ lie in the same plane? Explain.

Check the scalar triple product!

$\vec{a} \cdot (\vec{b} \times \vec{c})$

$\vec{b}$ $\langle 1, -1, 0\rangle$

$\times \vec{c}$ $\langle 7, 3, 2\rangle$

$$= \langle 2, 3, 1\rangle \cdot \langle -2, -2, 10\rangle = (-4 - 6 + 10) = 0$$

So yes, they are coplanar

(3)

4. Find the equation of the plane through $(1, 2, 3)$ that is normal to the vector $\langle 3, -1, 2\rangle$

point $(1,2,3)$

normal $\langle 3,-1,2\rangle$

plane: $3(x-1)-1(y-2)+2(z-3)=0$

(3)

5. Find the equation of the line through $(2, 1, 4)$ that is perpendicular to your plane from Question 4.

point $= (2,1,4)$

direction = normal to plane $= \langle 3,-1,2\rangle$

line:
$$x = 2+3t$$
$$y = 1-t$$
$$z = 4+2t$$

(3)

6. Find the equation of the plane that contains the points $(1,2,4)$, $(2,3,0)$, and $(2,1,2)$.

A B C

$\vec{AB} = \langle 1, 1, -4 \rangle$

$\times \vec{AC} = \langle 1, -1, -2 \rangle$

$\vec{n} = \langle -6, -2, -2 \rangle$

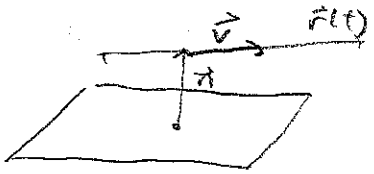
plane: $-6(x-1) - 2(y-2) - 2(z-4) = 0$

OR $3(x-1) + (y-2) + (z-4) = 0$

or $3x + y + z = 9$

(4)

7. Explain why the line $\mathbf{r}(t) = \langle 2 + 5t, 5 + t, 4 - 3t \rangle$ is parallel to the plane $2x - y + 3z = 2$. (Hint: Look at the relevant vectors for each object.)



$\vec{n}_{plane} = \langle 2, -1, 3 \rangle$

$\vec{v}_{line} = \langle 5, 1, -3 \rangle$

$\vec{n} \cdot \vec{v} = 10 - 1 - 9 = 0$

so $\vec{v} \perp \vec{n}$, $\vec{v} \parallel$ plane. line is parallel to the plane

(3)

8. (Bonus): You may have 0.5 or 1.5 points extra credit on this quiz. Which do you choose? (Note: If more than 25 percent of you choose one-and-a-half points, no one gets anything.)

My guess: 50% will chose 1.5 points