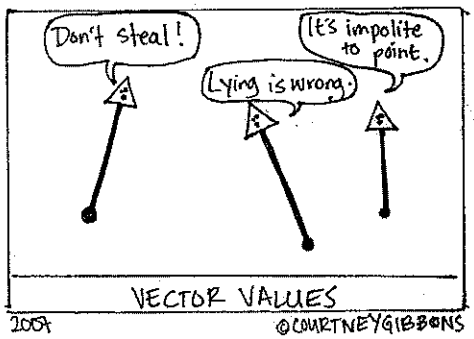
KEY

Math 225: Quiz the Third

This exam is closed book and closed notes. You may use your calculator for the purposes of arithmetic and for plotting equations, if helpful. When asked for specific values, however, you must show the relevant algebra. READ ALL DIRECTIONS CAREFULLY. You have the remainder of the period.



1. Consider the curve $\mathbf{r}(t) = \langle \cos(t), \sin(t), \cos^2(t) \rangle$. Determine any algebraic relationships present between the components. Plot the 2-dimensional view of the curve when viewed down the $x-$, $y-$ and $z-$axes, and give a (reasonable) 3-dimensional sketch of the curve.

$x = \cos t$
$y = \sin t$
$z = \cos^2 t$

① $x^2 + y^2 = 1$

② $z = x^2$

③ $z = 1 - y^2$

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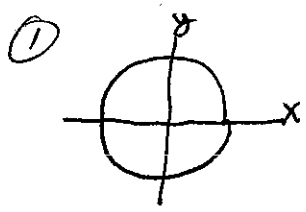
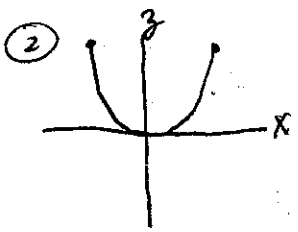
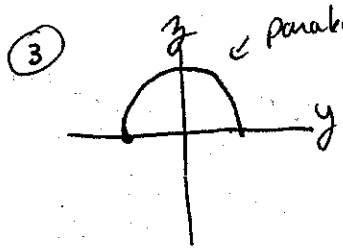
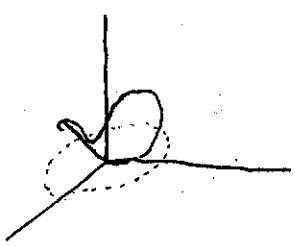






parabola, not semicircle

3D sketch



2. The trajectory of an object is given by the vector valued function

$$\mathbf{r}(t) = \langle t^2 + 1, \sqrt{1 - t^2}, \ln(t) \rangle$$

(a) For which values of t is the curve defined?

From $y \rightarrow -1 \leq t \leq 1$

From $z \rightarrow t > 0$

So, specifically $0 < t \leq 1$

(b) What values do x, y and z take on? $0 < t \leq 1 \Rightarrow$

$1 < x \leq 2$

$0 \leq y < 1$

$-\infty < z \leq 0$

(c) Find the speed of the object when $t = \frac{1}{2}$

$$\vec{r}\,'(t) = \left\langle 2t, \tfrac{1}{2}(1-t^2)^{-1/2} \cdot (-2t), \tfrac{1}{t} \right\rangle$$

$$= \left\langle 2t, \frac{-t}{\sqrt{1-t^2}}, \frac{1}{t} \right\rangle$$

$$\vec{r}\,'\left(\tfrac{1}{2}\right) = \left\langle 1, \frac{-\frac{1}{2}}{\sqrt{3/4}}, 2 \right\rangle$$

$$\left|\vec{r}\,'\left(\tfrac{1}{2}\right)\right| = \sqrt{1 + \frac{1}{3} + 4} = \frac{4}{\sqrt{3}}$$

3. Give a parametrization for the curve of intersection of $y^2 + z^2 = 1$ and $x + 2y + z = 3$.

$y^2+z^2=1 \rightarrow$ let $y = \cos t$, $z = \sin t$

$x+2y+z=3 \Rightarrow \quad x = 3-2\cos t - \sin t$

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(other variants one possible)

4. Determine the equation of the tangent line to

$$\mathbf{r}(t) = \langle t^2 + 2t, \sin(\pi t), \sqrt{2t-1} \rangle$$

when $t = 1$.

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$t=1 \rightarrow \vec{r}(1) = \langle 3, 0, 1 \rangle$

$\vec{r}\,'(t) = \langle 2t+2, \pi\cos(\pi t), \frac{1}{2}(2t-1)^{-1/2} \cdot 2 \rangle$

$\vec{r}\,'(1) = \langle 4, -\pi, 1 \rangle$

$\vec{\ell}(t) = \langle 3,0,1 \rangle + t\langle 4, -\pi, 1 \rangle$.

5. Suppose that the velocity of an object is given by $\mathbf{v}(t) = \langle 2t, t(t^2+1)^3, \sin(2t) \rangle$, and that the object is initially at the point (3, 1, 0). Find the vector valued function for ~~velocity~~ position, $\mathbf{s}(t)$.

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$\vec{v}(t) = \langle 2t, t(t^2+1)^3, \sin(2t) \rangle$

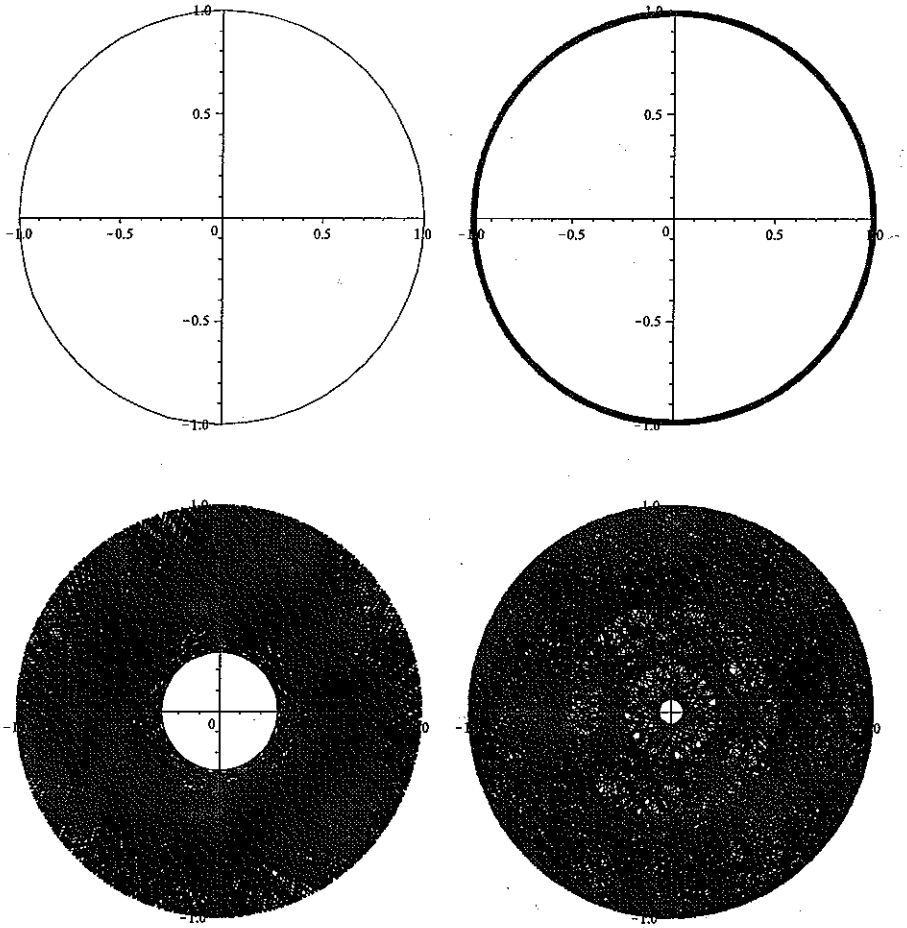
$\vec{s}(t) = \vec{s}(0) + \int_0^t \vec{v}(u)\,du$

$= \langle 3,1,0 \rangle + \int_0^t \langle 2u, u(u^2+1)^3, \sin(2u) \rangle du$

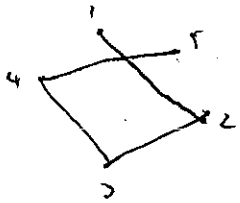
$= \langle 3,1,0 \rangle + \langle u^2, \frac{(u^2+1)^4}{8}, -\frac{\cos 2u}{2} \rangle \Big|_0^t = \langle 3,1,0 \rangle + \langle t^2, \frac{(t^2+1)^4}{8}, -\frac{\cos 2t}{2} \rangle + \langle 0, \frac{1}{8}, -\frac{1}{2} \rangle$

$= \langle t^2+3, \frac{(t^2+1)^4+1}{8}, \frac{1}{2} - \frac{\cos 2t}{2} \rangle$

6. (Extra Credit) Below are 4 different computer plots for the parametric equation $\mathbf{r}(t) = \langle \cos(t), \sin(t) \rangle$ for values of t ranging from 0 to 2π, 200π, 2000π and 20000π respectively. Explain why we might get different pictures for each of the graphs.



The computer draws the curve discretely by connecting the points for consecutive t values. As the range grows wider, the distance between consecutive values grows, leading to inaccuracies in the picture.



0, 2π 0, 2000π