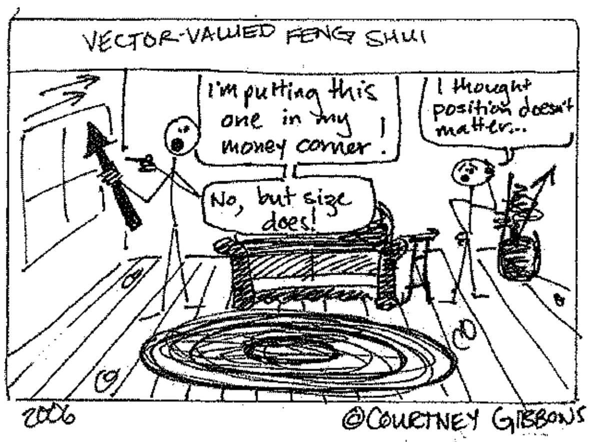
KEY

Math 225: Quiz the Fourth

This exam is closed book and closed notes. You may use your calculator for the purposes of arithmetic and for plotting equations, if helpful. When asked for specific values, however, you must show the relevant algebra. READ ALL DIRECTIONS CAREFULLY. You have the remainder of the period.



1. Find the arc length of

$$\mathbf{r}(t) = \left\langle \frac{t^2}{2}, \frac{2\sqrt{2}}{3}t^{\frac{3}{2}}, t \right\rangle$$

from $t = 1$ to $t = 2$.

$$\vec{r}\,'(t) = \langle t, \tfrac{\cancel{3}}{\cancel{2}}\cdot\tfrac{\cancel{2}}{\cancel{3}}\sqrt{2}\,t^{\frac{1}{2}}, 1 \rangle$$

4 $$|\vec{r}\,'(t)| = \sqrt{t^2+2t+1} = t+1$$

$$\int_1^2 t+1\,dt = \frac{t^2}{2} + t \Big|_1^2 = \frac{4}{2} + 2 - \left(\frac{1}{2}+1\right) = \frac{5}{2}$$

2. Set up, but don't compute, the integral for the length of $\mathbf{r}(t) = \langle \cos(t), \sin(3t), \sin(t) \rangle$

$[0, 2\pi)$

$\vec{r}'(t) = \langle -\sin t, 3\cos 3t, \cos t \rangle$

$$\int_0^{2\pi} \sqrt{\sin^2 t + 9\cos^2 3t + \cos^2 t}\, dt = \int_0^{2\pi} \sqrt{1 + 9\cos^2 3t}\, dt$$

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3. Find the curvature of

$$\mathbf{r}(t) = \left\langle \frac{t^2}{2}, \frac{2\sqrt{2}}{3} t^{\frac{3}{2}}, t \right\rangle$$

when $t = 1$.

$\vec{r}'(t) = \langle t, \sqrt{2}\, t^{1/2}, 1 \rangle$

$\vec{r}''(t) = \langle 1, \frac{\sqrt{2}}{2} t^{-1/2}, 0 \rangle$

$\vec{r}' \times \vec{r}'' = \langle -\frac{\sqrt{2}}{2} t^{-1/2}, 1, \frac{\sqrt{2}}{2} t^{1/2} - \sqrt{2}\, t^{1/2} \rangle$

$= \langle -\frac{\sqrt{2}}{2} t^{-1/2}, 1, -\frac{\sqrt{2}}{2} t^{1/2} \rangle$

4 $\vec{r}'(1) \times \vec{r}''(1) = \langle -\frac{\sqrt{2}}{2}, 1, -\frac{\sqrt{2}}{2} \rangle$

$$\frac{|\vec{r}'(1) \times \vec{r}''(1)|}{|r'(1)|^3} = \frac{\sqrt{\frac{1}{2} + 1 + \frac{1}{2}}}{(1+1)^3} = \frac{\sqrt{2}}{8}$$

4. Is the curvature in Question 3 defined when $t = 0$? Explain.

No, it is not, as $r''(t)$ is not defined ($t^{-1/2}$, in the second component, is not defined)

2

5. Let $\mathbf{r}(t) = \langle t^2, \frac{2}{3}t^3, t \rangle$.

(a) Calculate $\mathbf{T}(1)$. (Note: You NEED NOT find a general formula for $\mathbf{T}(t)$).

$r'(t) = \langle 2t, 2t^2, 1 \rangle$

$r'(1) = \langle 2, 2, 1 \rangle \qquad |r'(1)| = \sqrt{4+4+1} = 3$

$T(1) = \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle$

$2\frac{1}{2}$

(b) For this curve, $\mathbf{N}(1) = \langle \frac{-1}{3}, \frac{2}{3}, \frac{-2}{3} \rangle$. Find $\mathbf{B}(1)$.

$$B(1) = \begin{array}{l} T(1) = \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle \\ \underline{\times N(1)} = \langle \frac{-1}{3}, \frac{2}{3}, \frac{-2}{3} \rangle \end{array}$$

$$\langle -\frac{4}{9} - \frac{2}{9}, -\frac{1}{9} + \frac{4}{9}, \frac{4}{9} + \frac{2}{9} \rangle = \langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3} \rangle$$

$2\frac{1}{2}$

(c) Verify that $\mathbf{B}(1)$ is a unit vector.

1

$$|\vec{B}(1)| = \sqrt{\frac{4}{9} + \frac{1}{9} + \frac{4}{9}} = \sqrt{1} = 1$$

6. Again, let $\mathbf{r}(t) = \langle t^2, \frac{2}{3}t^3, t \rangle$.

(a) Calculate $\mathbf{v}(t)$ and $\mathbf{a}(t)$.

$\vec{v}(t) = \langle 2t, 2t^2, 1 \rangle$

2 $\quad \vec{a}(t) = \langle 2, 4t, 0 \rangle$

(b) Calculate a_T and a_N when $t = 1$.

$$a_T = \frac{r'(t) \cdot r''(t)}{|r'(t)|} = \frac{4t + 8t^3}{\sqrt{4t^2 + 4t^4 + 1}} \Big|_{t=1} \quad \frac{4+8}{\sqrt{9}} = \frac{12}{3} = 4$$

3 $$a_N = \frac{|r'(t) \times r''(t)|}{|r'(t)|} = \frac{|\langle -4t, 2, 4t^2 \rangle|}{\sqrt{4t^2 + 4t^4 + 1}} \Big|_{t=1} \quad \frac{|\langle -4, 2, 4 \rangle|}{\sqrt{9}} = \frac{\sqrt{36}}{\sqrt{9}} = \frac{6}{3} = 2$$

(c) Verify that $\mathbf{a}(1) = a_T \mathbf{T}(1) + a_N \mathbf{N}(1)$.

$a(1) = \langle 2, 4, 0 \rangle$

2 $\quad T(1) = \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle \qquad 4\,T(1) + 2\,N(1)$

$N(1) = \langle -\frac{1}{3}, \frac{2}{3}, -\frac{2}{3} \rangle \qquad = \langle \frac{8}{3}, \frac{8}{3}, \frac{4}{3} \rangle + \langle \frac{-2}{3}, \frac{4}{3}, \frac{-4}{3} \rangle =$

$\langle \frac{6}{3}, \frac{12}{3}, 0 \rangle = \langle 2, 4, 0 \rangle$!

✓, !!, etc.

7. (Bonus): What mathematical terms come from the Latin for:

(a) to kiss? osculating plane

(b) to touch? tangent whatever

(c) to throw alongside?

parabola (throw something & watch!)