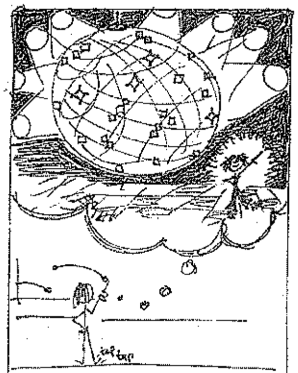
KEY

Math 225: Quiz the Fifth

This exam is closed book and closed notes. You may use your calculator for the purposes of arithmetic and for plotting equations, if helpful. When asked for specific values, however, you must show the relevant algebra. READ ALL DIRECTIONS CAREFULLY. You have the remainder of the period.



DISCOTINUNOUS FUNCTIONS HAVE THE MOST FUN

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1. Find the following limits, if they exist.

(a)

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+9}-3}$$

"$\frac{0}{0}$" $\to$ $\frac{\text{Top}}{\text{Bottom}} = \frac{2}{1}$ so we think the limit exists

$2\frac{1}{2}$

$$\frac{x^2+y^2}{\sqrt{x^2+y^2+9}-3} \cdot \frac{\sqrt{x^2+y^2+9}+3}{\sqrt{x^2+y^2+9}+3} = \frac{\cancel{(x^2+y^2)}\left(\sqrt{x^2+y^2+9}+3\right)}{\cancel{(x^2+y^2+9-9)}}$$

so limit $= \sqrt{9}+3 = 6$

(b)

$$\lim_{(x,y)\to(0,0)} \frac{y}{\sqrt{x^2+y^2}}$$

"$\frac{0}{0}$"

$2\frac{1}{2}$

let $y=0$, limit $=0$

let $y=x$, we get $\lim\limits_{x\to 0} \frac{x}{\sqrt{x^2+x^2}} = \frac{x}{\sqrt{2}\,x} = \frac{1}{\sqrt{2}} \neq 0$.

so limit DNE.

2. Consider the function $f(x,y) = \frac{1}{\sqrt{x^2+y^2}}$. $= (x^2+y^2)^{-\frac{1}{2}}$

(a) Find f_x and f_y.

$$f_x = -\frac{1}{2}(x^2+y^2)^{-3/2} \cdot (2x) = \frac{-x}{(\sqrt{x^2+y^2})^3}$$

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$$f_y = \frac{-y}{(\sqrt{x^2+y^2})^3}$$

(b) Find the equation to the tangent plane at (6, 8).

@ $(6,8)$ $\quad f = \frac{1}{\sqrt{6^2+8^2}} = \frac{1}{\sqrt{100}} = 0.1$

$f_x(6,8) = \frac{-6}{(10)^3} = -.006$

$f_y(6,8) = \frac{-8}{10^3} = -.008$

3

T.plane $\quad z = 0.1 - 0.006(x-6) - 0.008(y-8)$

(c) Use the tangent plane to approximate $f(6.02, 7.97)$. (no calculators, please!)

$$f(6.02, 7.97) \approx 0.1 - 0.006(6.02-6) - 0.008(7.97-8)$$

$$\approx (0.1 - 0.00012 + 0.00024)$$

2

$$= \underline{0.10024}$$

$\frac{\partial x}{\partial r} = \cos\theta \qquad \frac{\partial y}{\partial r} = \sin\theta$

3. Again, let $f(x,y) = \frac{1}{\sqrt{x^2+y^2}}$. Also, let $x = r\cos\theta$ and $y = r\sin\theta$. Using the chain rule, determine $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial \theta}$.

$\frac{\partial x}{\partial \theta} = -r\sin\theta \qquad \frac{\partial y}{\partial \theta} = r\cos\theta$

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r}$$

$$= \frac{-x}{(\sqrt{x^2+y^2})^3}\cdot\cos\theta + \frac{-y}{(\sqrt{x^2+y^2})^3}\cdot\sin\theta$$

3

$$= \frac{-r\cos^2\theta}{r^3} - \frac{r\sin^2\theta}{r^3}$$

$$= -\frac{1}{r^2}$$

$$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \theta}$$

$$= \frac{-x}{(\sqrt{x^2+y^2})^3}\cdot -r\sin\theta + \frac{-y}{(\sqrt{x^2+y^2})^3}\, r\cos\theta$$

$$= \frac{r^2\sin\theta\cos\theta - r^2\sin\theta\cos\theta}{r^3} = 0$$

4. Wheat production (W) is dependent on temperature (T) and rainfall (R). It is estimated that temperature is increasing at a rate of 0.05° C/year and rainfall is decreasing at a rate of 0.1 cm/year. It is also estimated that, at current levels of production, $\frac{\partial W}{\partial T} = -2$ and $\frac{\partial W}{\partial R} = 8$.

(a) Give a prose description of the significance of the signs of the partial derivatives $\frac{\partial W}{\partial T}$ and $\frac{\partial W}{\partial R}$.

$\frac{\partial W}{\partial T} = -2$ As temperature increases, wheat production decreases

$\frac{\partial W}{\partial R} = 8$ As rainfall increases, wheat production increases

2

(b) Using these estimates, find $\frac{dW}{dt}$.

$$\frac{dW}{dt} = \frac{\partial W}{\partial T}\frac{dT}{dt} + \frac{\partial W}{\partial R}\frac{dR}{dt}$$

2

$$= (-2)(0.05) + 8(\bar{0}.1) = \underline{-0.9}$$

F_z

5. Consider the equation $x^2 + y^2 + xyz = 1$.

   (a) When can we solve for z as a function of x and y? (Be as specific as possible, in terms of x, y and z.)

   We can solve for z in terms of x and y provided $F_z = xy \neq 0$

   So when neither x nor y is 0

2

   (b) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

   $$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z} = \frac{-(2x+yz)}{xy} \qquad \frac{\partial z}{\partial y} = \frac{-F_y}{F_z} = \frac{-(2y+xz)}{xy}$$

3

6. Extra Credit:

   (a) What is your birthday? (Month and Day only. No years. Please.) November 19

   (b) Of the 58 of us in the two classes, what is the approximate probability that at least 2 of us have the same birthday?

   i. 16 % ii. 32 % iii. 80 % iv. 99 % +