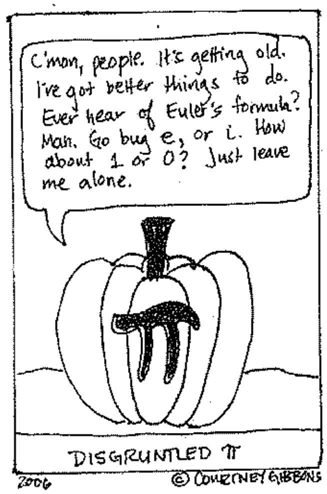
KEY

Math 225: Quiz the Sixth
October 27, 2009

You know the drill by now. No books, no notes, no colleagues, and no answers without justification.
READ ALL QUESTIONS CAREFULLY



1. Let $f(x, y)$ be a function of two variables. Give formulas for each of the following.

   (a) ∇f

   $$\vec{\nabla} f = \langle f_x, f_y \rangle$$

   (b) $D_{\mathbf{u}} f$

   $$D_{\vec{u}} f = \vec{\nabla} f \cdot \vec{u}$$

   (c) The discriminant, D.

   $$D = f_{xx} f_{yy} - (f_{xy})^2$$

2. Let $f(x, y) = 3x^2 + 2xy^2 + y^4$.

(a) Find $D_{\mathbf{u}}f$ at the point $(1, 3)$ in the direction $\langle 3, 2\rangle$.

$\langle 3,2\rangle \xrightarrow{\vec{u}} \langle \frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \rangle$

$\vec{\nabla} f = \langle 6x + 2y^2, 4xy + 4y^3 \rangle$

$\vec{\nabla} f \big|_{(1,3)} = \langle 6 + 18, 12 + 108 \rangle$

$\langle 24, 120 \rangle$

$D_{\vec{u}} f = \langle 24, 120 \rangle \cdot \langle \frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \rangle = \frac{72}{\sqrt{13}} + \frac{240}{\sqrt{13}} = \frac{312}{\sqrt{13}}$

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(b) Find $D_{\mathbf{u}}f$ at the point $(1, 3)$ in the direction for which that derivative is a minimum.

minimum of $D_{\vec{u}} f$ occurs antiparallel to $\vec{\nabla} f$, so

direction $= \langle -24, -120 \rangle$

magnitude $= \sqrt{24^2 + 120^2} = 12\sqrt{2^2 + 10^2} = 12\sqrt{104}$
$= 24\sqrt{26}$

$D_u f = -24\sqrt{26}$

3. Let $x^2y^2 + y^2z^2 + x^2z^2 = 84$. Find the equation of the tangent plane to this surface at the point $(1, -2, 4)$.

Point $(1,-2,4)$

Normal $\vec{n} = \vec{\nabla} F = \langle 2xy^2 + 2xz^2, 2yx^2 + 2yz^2, 2zx^2 + 2zy^2 \rangle \big|_{(1,-2,4)}$

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$= \langle 2(1)(4+16), 2(-2)(1+16), 2(4)(1+4) \rangle$

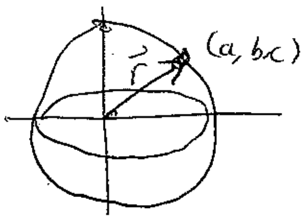
$= \langle 40, -68, 40 \rangle$

Plane: $40(x-1) - 68(y+2) + 40(z-4) = 0$

$10(x-1) - 17(y+2) + 10(z-4) = 0$

4. Consider the sphere $x^2 + y^2 + z^2 = r^2$.

(a) Express the radius of the sphere from the origin to (a, b, c) as a vector.



$\vec{r} = \langle a, b, c \rangle$

(b) Find the vector normal to the tangent plane to sphere at (a, b, c)

$\vec{n} = \vec{\nabla} F = \langle 2x, 2y, 2z \rangle \big|_{(a,b,c)} = \langle 2a, 2b, 2c \rangle$

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(c) What do you notice, geometrically?

$\vec{n}$ is parallel to $\vec{r}$, that is, $\vec{r}$ is perpendicular to the tangent.

5. Let $f(x, y) = e^{x^2} \cos(x^3) + e^y \ln(\sin(\sqrt{y}))$. Find f_{xy} (hint: you need not show any intermediate steps!).

$f_{xy} = 0$

$f(x,y) = g(x) + h(y)$

$f_x = g'(x) + 0$

$f_{xy} = 0 + 0$

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6. Find and classify all critical points of $f(x, y) = x^2 + 2xy + \frac{y^3}{3} - 8y$ (there are two of them).

(4)

$f(x,y) = x^2 + 2xy + \frac{y^3}{3} - 8y$

$f_x = 2x + 2y = 0$

$f_y = 2x + y^2 + (-8) = 0$

$2y = y^2 - 8$

$y^2 - 2y - 8 = 0$; $(y-4)(y+2) = 0$

$y = 4$; $y = -2$

$\Rightarrow x = -4$; $x = 2$

pts : $(2,-2)$ $(-4,4)$

$D = f_{xx} f_{yy} - (f_{xy})^2$

$= 2(2y) - (2)^2$

$= 4y - 4$

$D(2,-2) = -8-4 < 0$ $(2,-2)$ is a saddle point

$D(-4,4) = 16-4 > 0$ $(-4,4)$ is a minimum

$f_{xx} > 0$