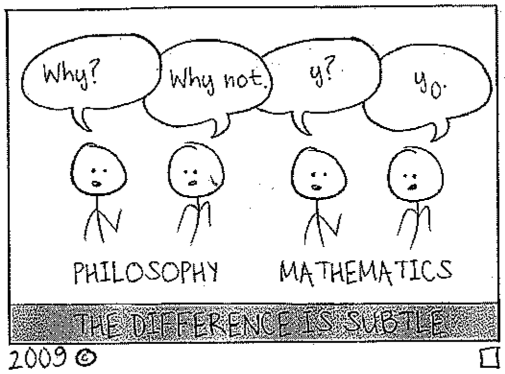
KEY

**Math 225: Quiz the ~~Sixth~~ Seventh**

November 4, 2009

You know the drill by now. No books, no notes, no colleagues, and no answers without justification.
**READ ALL QUESTIONS CAREFULLY**




1. Using Lagrange multipliers, find the maximum value of $f(x, y) = 4xy$ subject to $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

$f = 4xy \qquad g = \frac{x^2}{16} + \frac{y^2}{9} = 1$

$\vec{\nabla} f = \lambda \vec{\nabla} g$

$4y = \frac{2x}{16}\lambda$ mult by x

$4x = \frac{2y}{9}\lambda$ mult by y

$4xy = \frac{\cancel{2}x^2}{16}\cancel{\lambda} = \frac{2y^2}{9}\cancel{\lambda}$

$\frac{x^2}{16} = \frac{y^2}{9} = \frac{1}{2} \qquad x = \sqrt{8} \;;\; y = \frac{3}{\sqrt{2}}$

$f = 4 \cdot \sqrt{8} \cdot \frac{3}{\sqrt{2}} = \boxed{24}$

2. Let $f(x,y) = xy + \frac{64}{x} + \frac{64}{y}$

(a) Find the only critical point for which the function is defined.

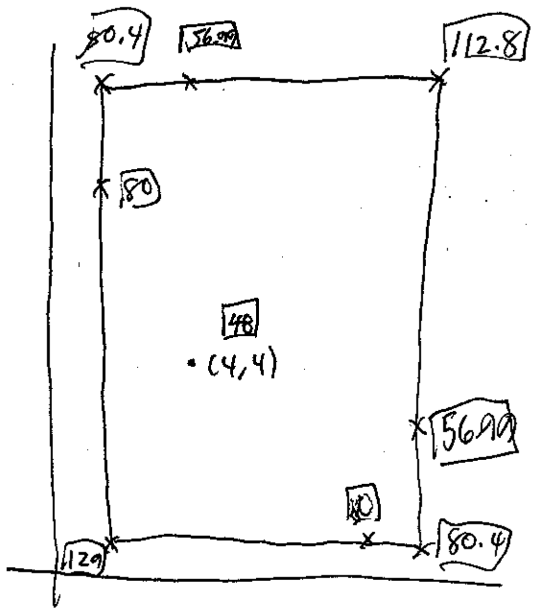
$f_x = y - \frac{64}{x^2} = 0 \qquad y = \frac{64}{x^2}$

$f_y = x - \frac{64}{y^2} = 0$

$f_y = x - \frac{64}{\left(\frac{64}{x^2}\right)^2} = x - \frac{x^4}{64} = x\left(1 - \frac{x^3}{64}\right) = 0 \qquad x = 4;\ y = \frac{64}{16} = 4$

$(4,4)$

(b) Find the global maximum and global minimum for $f$ on $1 \le x \le 10$ and $1 \le y \le 10$ (Hint: Use symmetry here to do only two, rather than four, boundary segments).




$x = 1 \to f = y + \frac{64}{y} + 64$

$f' = 1 - \frac{64}{y^2} = 0 \quad y = 8$

test $1,\ 8,\ 10$

$f = 129,\ 80,\ 80.4$

or $y = 1$, same

$x = 10 \quad f = 10y + \frac{64}{y} + 6.4$

$f' = 10 - \frac{64}{y^2} = 0$

$G_{max} = 129$

$G_{min} = 48$

$y^* = \sqrt{6.40} = \cancel{8\sqrt{10} > 10}$, ~~so no crit point.~~

$y = \sqrt{6.4}$

$10\sqrt{6.4} + \frac{64}{\sqrt{6.4}} + 6.4$

$25.5 + 25.09 + 6.4$

3. Do TWO of the following problems. If you choose to work on all three, please CLEARLY INDICATE WHICH TWO you are submitting for credit. For all of these, you may use any method that you wish.

   (a) Find the point(s) on $x^2y = 1$ closest to the origin.

   (b) The plane $3x + 5y + z = 9$ cuts a region in the first octant. Find the volume of the largest box, with sides parallel to the axes, that can fit in this region.

   (c) A cylindrical container is to be mailed. The sum of the height and circumference cannot exceed 108 inches. What is the largest volume that can be mailed?

(a) minimize

$x^2+y^2$ subj to $x^2y=1$

So $x^2 = \frac{1}{y}$

$f = \frac{1}{y}+y^2 \quad f' = \frac{-1}{y^2}+2y=0 \quad \frac{1}{y^2}=2y \quad y^3=\frac{1}{2} \quad y=\sqrt[3]{1/2} = 2^{-\frac{1}{3}}$

$x^2 = \frac{1}{y} = 2^{+1/3}$

$x = \pm 2^{+1/6}$

points $(\pm 2^{1/6}, 2^{1/3})$

(b) minimize

$V = xyz$ subj to $\underset{P}{3x+5y+z=9}$

$V_x = yz = 3\lambda = \lambda P_x$

$V_y = xz = 5\lambda \quad \lambda P_y$

$V_z = xy = \lambda \quad \lambda P_z$

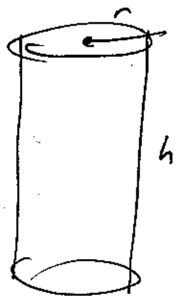
$3x\lambda = 5y\lambda = z\lambda \qquad 3x=5y=z$

$9x=9 \quad x=1; y=\frac{3}{5}, z=3$

$V = \frac{9}{5}$

More space for (3):

C.




$V = \pi r^2 h$ subj to $h + 2\pi r = 108$

$h = 108 - 2\pi r$

$V = \pi r^2 (108 - 2\pi r)$

$V = 108\pi r^2 - 2\pi^2 r^3$

$V' = 216\pi r - \cancel{8}6\pi^2 r^2 = 0 \qquad 216\pi r - 6\pi^2 r^2$

$216 \pi r = 6 \pi^2 r^2$

$\cancel{r = 27/\pi} \qquad \cancel{36/\pi} \qquad r = 36/\pi$

$\cancel{h = 108 - 54\sqrt{\pi}} \qquad h = 108 - \frac{2\pi \cdot 36}{\pi}$

$= 108 - 72 = 54$

36.

$V = \frac{36^3}{\pi}$

4. Extra Credit: Pretend to toss a coin 5 times. Record your results as a list of heads and tails.

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