KEY

Math 225: Quiz the Last

December 9, 2009

You know the drill by now. No books, no notes, no colleagues, and no answers without justification.
READ ALL QUESTIONS CAREFULLY

1. Set up the bounds for

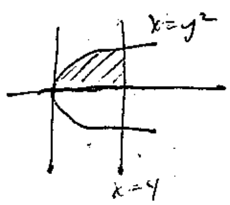
$$\iiint_E f(x,y,z)\,dV$$

where E is bound below by $z=0$, above by $z=y$ and by the parabolic cylinder $x=y^2$ and the plane $x=4$.

$0 \le z \le y$

$y^2 \le x \le 4$

$0 \le y \le 2$



$y \ge 0$

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$$\int_0^2 \int_{y^2}^4 \int_0^y f(x,y,z)\,dz\,dx\,dy$$

2. Convert

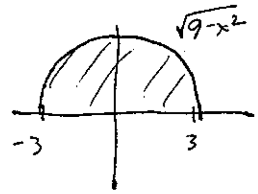
$$\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}}\int_{0}^{x^2+y^2} x^2+y^2+z^2\,dz\,dy\,dx$$

to cylindrical coordinates.

$0 \le z \le r^2$

$0 \le r \le 3$

$0 \le \theta \le \pi$



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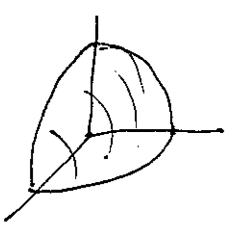
$$\int_0^{\pi}\int_0^{3}\int_0^{r^2} (r^2+z^2)\, r \, dz\, dr\, d\theta$$

3. Find

$$\iiint_E \sqrt{x^2+y^2+z^2}\, dV$$

where E is the portion of the unit sphere (centered at the origin) in the first octant.



$0 \le \rho \le 1$

$0 \le \theta \le \frac{\pi}{2}$

$0 \le \phi \le \frac{\pi}{2}$

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{1} \rho \cdot \rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$$

$$= \int_0^{\pi/2}\int_0^{\pi/2} \frac{1}{4} \sin\phi \, d\theta\, d\phi$$

$$= \int_0^{\pi/2} \frac{\pi}{8} \sin\phi \, d\phi$$

$$= \left[-\cos\phi\right]_0^{\pi/2} \cdot \pi/8 = \pi/8$$

4

4. Find $\int_C x^2 + y^2\, ds$ where C is the line segment from $(0,1)$ to $(4,4)$.

$C:$ $x = 4t$, $y = 1+3t$, $0 \le t \le 1$ $\qquad ds = \sqrt{16+9} = 5$

$$\int_0^1 (4t)^2 + (1+3t)^2 \cdot 5\, dt$$

$$= 5\int_0^1 16t^2 + 9t^2 + 6t + 1\, dt$$

$$= 5\left[\frac{25t^3}{3} + 3t^2 + t\right]_0^1 = 5\left[\frac{25}{3} + 3 + 1\right] = \frac{5 \cdot 37}{3} = \frac{185}{3}$$

4

5. Let $\mathbf{F} = \langle 2xy, y^2 - x^2 \rangle$. Find the work done in moving along the path $\langle t, t^2 \rangle$ from $t = 2$ to $t = 4$.

$$\int_C \vec{F} \cdot d\vec{r} = \int_C 2xy\, dx + (y^2 - x^2)\, dy$$

$$= \int_2^4 2t^3 \cdot 1\, dt + (t^4 - t^2)(2t)\, dt$$

$$= \int_2^4 2t^3 + 2t^5 - 2t^3\, dt = \frac{2t^6}{6}\Big|_2^4 =$$

$$\frac{\cancel{2}(4096)}{3} - \frac{64}{3} = \frac{4032}{3}$$

$$= \boxed{1344}$$

4

$P \qquad Q$

6. Find a function $f(x, y)$ such that $\nabla f = \langle x^2 + 2xy, x^2 + y^2 \rangle$.

Check: $Q_x = 2x = P_y$ ok!

$f_x = x^2 + 2xy$

$f = \frac{x^3}{3} + x^2 y + g(y)$

$f_y = 0 + \cancel{2x} x^2 + g'(y) = x^2 + y^2$

$g'(y) = y^2$

$g = y^3/3$

$\longrightarrow f = \frac{x^3}{3} + x^2 y + \frac{y^3}{3}$

$2\frac{1}{2}$

7. Use your function to find $\int_C (x^2 + 2xy)\, dx + (x^2 + y^2)\, dy$ where C is any path from $(0, 1)$ to $(2, 3)$.

$\int_C \vec{\nabla} f \cdot dr = \frac{x^3}{3} + x^2 y + \frac{y^3}{3} \Big|_{(0,1)}^{(2,3)}$

$= \frac{8}{3} + 12 + 9 - \left[0 \cdot 0 + \frac{1}{3}\right]$

$24\frac{1}{3}$

$2\frac{1}{2}$

8. (Bonus) You may have one or two points extra credit. There is someone else in the class with the same numbered quiz as yours.

- If you both opt for 1 point, you both get 1 point.
- If only one of you opts for 2 points, that person gets two points, while the other gets nothing.
- If both of you opt for 2 points, neither gets anything.