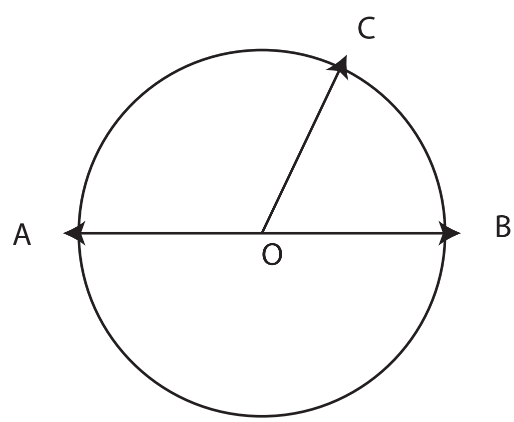
Math 225: Practice Exam 1t

This exam is meant to cover the types of problems that you might encounter on Friday's Midterm. It is by no means exhaustive. To get the most out of your studies, you should also review our past quizzes, homeworks, and class notes. This exam will serve as the basis of our review on Wednesday.

1. Give parametric equations for the line segment between the points $(1,3)$ and $(2,-1)$.
2. Find the points at which the curve $r(t) = \langle t^2, t^3 - 3t \rangle$ has a horizontal tangent.
3. Find the area inside the top half of the cardioid $r = 1 + \cos(\theta)$.
4. Let $A = (2,3,1)$, $B = (6,1,5)$, and let $P = (x,y,z)$
   (a) Find the distance from A to B.
   (b) Write the following statement as an equation:
   The distance from P to A is the same as the distance from P to B.
   (c) Simplify your equation and describe the set of points (x,y,z) that satisfy the description in part (b).
5. Consider the following diagram:



In this picture, A and B are on opposite sides of a diameter, C is any other point on the circle, and O is the center. Prove, using vectors, that angle ACB is a right angle.

6. Consider the following two line equations:

$$\ell_1(t) = \langle 1+3t, t, 2-2t \rangle; \ \ell_2(t) = \langle -6t+3, -2t+3, 4t \rangle$$

   (a) Explain why ℓ_1 is parallel to ℓ_2.
   (b) Find the plane that contains ℓ_1 and ℓ_2.

7. Suppose that we have three lines, each of which is skew to the other two. Might there be a plane parallel to all 3 lines? Must there be a plane parallel to all 3 lines? Explain.

8. Consider the equation $\mathbf{r}(t) = \langle \cos(t), \sin(t), \cos(2t) \rangle$.

   (a) This curve is the intersection of which two surfaces? (Here, you'll need algebraic relations between x, y and z components. Try a trig identity for the relation with z, which I can 'sell' to you if necessary.) Be sure that you give the names of the surfaces as well as the algebraic equations.

   (b) Give a rough sketch of the curve. (You may give a prose description to aid you here.)

   (c) Find the equation of the tangent line to $\mathbf{r}(t)$ when $t = \frac{\pi}{2}$.

9. Suppose that the position of an object is given by $\mathbf{r}(t) = \langle t^2, e^t, te^t \rangle$.

   (a) Find the velocity and acceleration of the particle as a function of time.

   (b) Find the speed of the particle as a function of time. Is the speed ever 0?

   (c) Set up, but don't compute, the integral for the distance that the particle travels from $t = 1$ to $t = 2$.

10. Match the equation to the surface description. Warning: There is one outlier in each group!!

(a) $x^2 + 9y^2 + 81z^2 = 81$	I Hyperboloid of One Sheet
(b) $x^2 + 9y^2 + 81 = 81z^2$	II Hyperboloid of Two Sheets
(c) $x^2 + 9y^2 + 81 = 81z$	III Ellipsoid
(d) $x^2 - 9y^2 + 81 = 81$	IV Elliptical Paraboloid

11. For vectors $\mathbf{x}, \mathbf{y}$ and $\mathbf{z}$, we have that

$$\mathbf{x} \times (\mathbf{y} \times \mathbf{z}) = (\mathbf{x} \cdot \mathbf{z})\mathbf{y} - (\mathbf{x} \cdot \mathbf{y})\mathbf{z}$$

Use this formula and the definition of the binormal vector, $\mathbf{B}(t)$, to prove that $\mathbf{N}(t) \times \mathbf{B}(t) = \mathbf{T}(t)$.

12. (Bonus) Prove that if $\mathbf{r}(t)$ has zero curvature for all t, then $\mathbf{r}(t)$ is either a straight line or a single point.