Practice Exam 1t
Odd #s

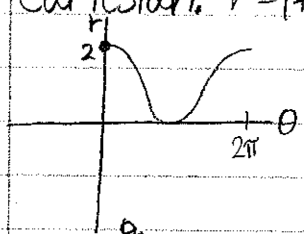
1. line segment (1,3) and (2,−1)

$x = x_0 + \Delta x t = 1 + (2-1)t = 1+t$
$y = y_0 + \Delta y t = 3 + (-1-3)t = 3-4t$

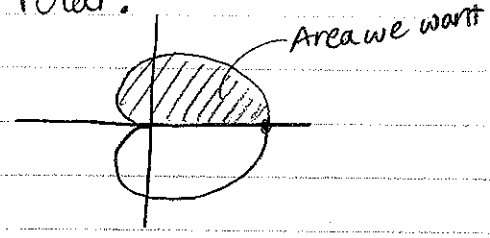
$0 \le t \le 1$ because it's a line segment

3. $r = 1+\cos\theta$

Cartesian: $r = 1+\cos\theta$



Polar:



$$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2\,d\theta = \frac{1}{2}\int_0^{\pi}(1+\cos\theta)^2\,d\theta$$

$$= \frac{1}{2}\int_0^{\pi}(1+2\cos\theta+\cos^2\theta)\,d\theta$$

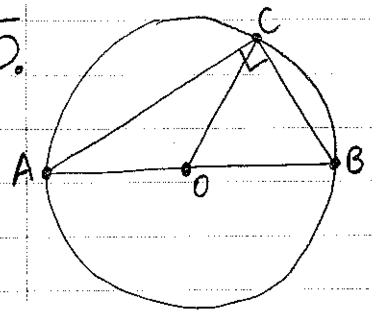
$$= \frac{1}{2}\int_0^{\pi}\left(1+2\cos\theta+\frac{1+\cos 2\theta}{2}\right)d\theta$$

$$= \frac{1}{2}\left[\theta+2\sin\theta+\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right]_0^{\pi}$$

$$= \frac{1}{2}\left[\pi+0+\frac{\pi}{2}+0-0\right]$$

$$= \frac{1}{2}\left(\frac{3\pi}{2}\right)$$

$$= \frac{3\pi}{4}$$

5. 

Prove that $\angle ACB$ is a right angle.

$\overrightarrow{CA} \perp \overrightarrow{CB}$, so the dot product should equal zero.

$$\overrightarrow{CA} \cdot \overrightarrow{CB}$$
$$(\overrightarrow{OA} - \overrightarrow{OC}) \cdot (\overrightarrow{OB} - \overrightarrow{OC}) \qquad \overrightarrow{OB} = -\overrightarrow{OA}$$
$$(\overrightarrow{OA} - \overrightarrow{OC}) \cdot (-\overrightarrow{OA} - \overrightarrow{OC})$$
$$-\overrightarrow{OA} \cdot \overrightarrow{OA} + \cancel{\overrightarrow{OC} \cdot \overrightarrow{OA}} - \cancel{\overrightarrow{OC} \cdot \overrightarrow{OA}} + \overrightarrow{OC} \cdot \overrightarrow{OC}$$
$$-|\overrightarrow{OA}|^2 + |\overrightarrow{OC}|^2 = 0$$

Since $\overrightarrow{OA}$ and $\overrightarrow{OC}$ are radii, $|\overrightarrow{OA}| = |\overrightarrow{OC}|$, so $-|\overrightarrow{OA}|^2 + |\overrightarrow{OC}|^2 = 0$

7. Yes, there might be a plane parallel to all three lines. The three lines would be perpendicular to the plane's normal vector. (Imagine signpost pointing to multiple cities)

There isn't always a plane parallel to all three lines. If we take 3 skew lines, with each one parallel to one of the coordinate axes (one parallel to x-axis, one parallel to y-axis, one parallel to z-axis), there is no plane parallel to all 3 lines.

9\. a) $\vec{r}(t) = \langle t^2, e^t, te^t \rangle$

$\vec{v}(t) = \vec{r}\,'(t) = \langle 2t, e^t, te^t + e^t \rangle$

$\vec{a}(t) = \vec{r}\,''(t) = \langle 2, e^t, te^t + 2e^t \rangle$

b) Speed: $|\vec{r}\,'(t)| = \sqrt{4t^2 + e^{2t} + (te^t + e^t)^2}$

Does the speed ever equal zero?

$$\sqrt{\underbrace{4t^2}_{\text{zero or +}} + \underbrace{e^{2t}}_{\text{always +}} + \underbrace{(te^t + e^t)^2}_{\text{zero or +}}}$$

Speed never equals zero, since $e^{2t} > 0$

c) $\int_1^2 \sqrt{4t^2 + e^{2t} + (te^t + e^t)^2}\, dt$

11\. $\vec{x} \times (\vec{y} \times \vec{z}) = (\vec{x} \cdot \vec{z})\vec{y} - (\vec{x} \cdot \vec{y})\vec{z}$ Prove $\vec{N}(t) \times \vec{B}(t) = \vec{T}(t)$

Definition of $\vec{B}(t)$:

$\vec{B} = (\vec{T} \times \vec{N})$

$\vec{N} \times \vec{B} = \vec{N} \times (\vec{T} \times \vec{N})$

$= \underbrace{(\vec{N} \cdot \vec{N})}_{1}\vec{T} - \underbrace{(\vec{N} \cdot \vec{T})}_{0}\vec{N}$ using given equation for triple cross product

because $\vec{N}$ is a unit vector → 1 0 ← because they are orthogonal

$= \vec{T}$

Even #'s

2. $r(t) = \langle t^2, t^3-3t \rangle$

$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{3t^2-3}{2t}$

horizontal tangent $\frac{dy}{dx} = 0$

$3t^2-3=0 \quad t=\pm 1$

Points: $t=1 \quad (1^2, 1^3-3(1)) = (1,-2)$

$t=-1 \quad ((-1)^2, (-1)^3-3(-1)) = (1,2)$

4. $A=(2,3,1) \qquad B=(6,1,5) \qquad P=(x,y,z)$

a) $d = \sqrt{(6-2)^2+(3-1)^2+(5-1)^2} = \sqrt{16+4+16} = 6$

b) $\sqrt{(x-2)^2+(y-3)^2+(z-1)^2} = \sqrt{(x-6)^2+(y-1)^2+(z-5)^2}$

c) $x^2-4x+4+y^2-6y+9+z^2-2z+1 = x^2-12x+36+y^2-2y+1+z^2-10z+25$

$8x-4y+8z=48$

↳ equation for a plane

$2x-y+2z=12$

6.a $L_1 = \langle 1+3t, t, 2-2t \rangle \qquad L_2 = \langle -6t+3, -2t+3, 4t \rangle$

$\vec{v}_1 = \langle 3,1,-2 \rangle \qquad \vec{v}_2 = \langle -6,-2,4 \rangle$

$\vec{v}_2 = -2\vec{v}_1$ → lines are parallel

b. point: $(1,0,2) \qquad (3,3,0)$ ← point on each line

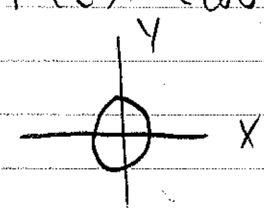
normal: $\langle 3,1,-2 \rangle$

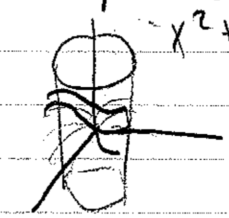
$\times \langle 2,3,-2 \rangle$

$\vec{n} = \langle 4,2,7 \rangle$

plane: $4(x-1)+2(y-0)+7(z-2)=0$

8. $\vec{r}(t) = \langle \cos t, \sin t, \cos 2t \rangle$





$x^2+y^2=1$ → cylinder

$\cos 2t = \cos^2 t + \sin^2 t$

$z = x^2-y^2$ → hyperbolic paraboloid

$\vec{r}(t)$ when $t=\frac{\pi}{2}$

$\vec{r}'(t) = \langle -\sin t, \cos 2t, 2\sin 2t \rangle$

$\vec{r}'(\frac{\pi}{2}) = = \langle -1, 0, 0 \rangle$

line: $\langle 0,1,-1 \rangle + \langle -1,0,0 \rangle t$

$l(t)$

10 a) $x^2+9y^2+81z^2=81$ → Ellipsoid III

b) $x^2+9y^2+81=81z^2$ → Hyperboloid 2 sheets II

c) $x^2+9y^2+81=81z$ → Elliptical paraboloid IV

d) $x^2-9y^2+81=81$ → none of the above

$x^2=9y^2$

$x=\pm 3y$

12. $\kappa = \dfrac{T'(t)}{|r'(t)|} = 0$

$|\vec{T}'(t)|=0$

$\vec{T}'(t) = \langle 0,0,0\rangle$

$\vec{T}(t) = \langle a,b,c\rangle$

$\vec{r}'(t) = \lambda\langle a,b,c\rangle$

$= \langle a', b', c'\rangle$

$\vec{r}(t) = \langle a't+d, b't+e, c't+f\rangle$