Math 225: Practice Exam the Second

Topics covered since the last exam:

- Graphs of functions $z = f(x, y)$
- Level Curves
- Limits of Multivariable Functions
- Partial Derivatives
- Second- and Higher-Order Derivatives and Clairaut's Theorem
- The Tangent Plane and Approximations of Functions
- The Chain Rule
- The Implicit Function Theorem
- Tangent Planes of Implicitly Defined Functions
- The Gradient
- Directional Derivatives and Maximum Rates of Change
- Critical Points and their Classification
- Bounded Optimization
- Constrained Optimization
- Lagrange Multipliers
- Basic Multiple Integration
- Integration as a Volume in 3D
- Integration with variable limits

Here are some problems that are representative of the types that will be seen on Wednesday's exam. Please note that this set is LONGER than you'll actually see on the test. I have tried to be as inclusive as possible of the types of questions that you might see.

1. (a) Draw the level curves to $f(x, y) = \frac{y}{x^2}$ for $f(x, y) = -2, -1, 0, 1, 2$.
   (b) What is happening with these level curves at the origin?
2. Find
$$\lim_{(x,y)\to(0,0)} \frac{(x-y)^2}{x^2+y^2}$$
   if it exists.

3. Find, if it exists, the following limit

$$\lim_{(x,y)\to(0,0)} \frac{\cos(x^2+y^2)-1}{(x^2+y^2)^2}$$

4. Suppose that for a continuous and differentiable function $f(x,y)$, we know that $f_x(x,y) = 2xy + x^3$. Give at least three possible functions that could be $f_y(x,y)$.

5. Consider the function $f(x,y) = x^4y - x^2y^3$
   (a) Find the direction in which f *decreases* the fastest from the point $(2,-3)$.
   (b) Find the tangent plane and use it to approximate $f(2.02,-2.97)$.
   (c) Consider the level curve $f(x,y) = 60$.
      i. Find $\frac{dy}{dx}$ for this curve.
      ii. Evaluate $\frac{dy}{dx}$ at $(2,-3)$. How does this value relate to your answer from part (a)?

6. Let $f(x,y,z) = xy + z^2$ and let $\langle x,y,z\rangle = \langle t,t^2,t^4\rangle$. Calculate $\frac{df}{dt}$ when $t = 1$.

7. The length, width and height are changing on a rectangular box. The height is increasing at a rate of 1 in/min, the width is decreasing at a rate of .5 in/min, and the length is increasing at a rate of 2 in/min. How fast is the volume of the box changing when the length is 3, the width is 4, and the height is 5? How fast is the surface area changing at this time?

8. Find and classify the critical points of $f(x,y) = \frac{x^3}{3} + \frac{y^3}{3} - \frac{x^2}{2} - y^2$. (There are four of them.)

9. Find the minimum surface area of a box without a top that has volume 27.

10. Find the maximum volume of a box in the first octant with opposite corners at the origin and on the plane $x + 2y + 3z = 10$, respectively. (You may use any method that you wish).

11. Consider the points $X = (x,y)$ and $B = (0,b)$. Find the minimum distance from X to the line $y = a$, and then find the set of points X that are the same distance from B and this line. What familiar equation is this?

12. Find the following integrals. Where necessary or appropriate, you may reverse the order of integration.
   (a)
   $$\int\int_R yx^2 + xy + y^2 dy\, dx, \text{ where } R = [0,2]\times[0,4]$$
   (b)
   $$\int_0^1 \int_0^2 y\cos(xy) dy\, dx$$
   (c)
   $$\int\int_R x + 2y\, dA$$
   where R is bound by $y = x^2$ and $y = 8\sqrt{x}$