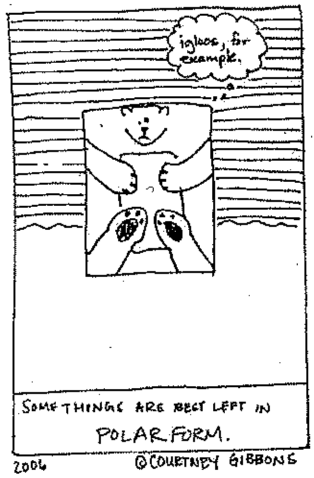
KEY

Math 225: Quiz the First
September 9, 2011

This quiz is closed book and closed notes. You may not use a calculator on this quiz. Please justify all of your answers. You have until 5 minutes before the hour to finish.



1. Suppose $(x(t), y(t))$ are a set of parametric equations for $a \le t \le b$. To reverse the direction of the parametric curve, what algebraic operation do we need to perform?

2

"run time backwards".

i.e. $\to x(-t), y(-t)$ for $-b \le t \le -a$

Replace "t" with "$-t$"

2. What (if any) is the difference between the plots of the line $y = 1 - x$ and the parametric curve $(x = \cos(t), y = 1 - \cos(t))$?

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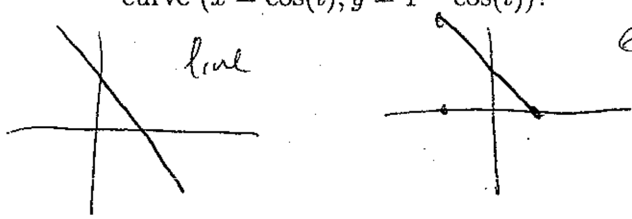


← oscillating line segment

3. Consider the point $(1, \frac{\pi}{3})$ in polar coordinates. Give two other coordinate pairs that give the same point, one of which has a negative r value.

2

$(1, \frac{\pi}{3}) = (1, \frac{7\pi}{3}) = (-1, \frac{4\pi}{3})$ for example.

4. Consider the parametric equations $x = 1 - t^2$, $y = t^3 - t$.

(a) For which value of t does the curve go through the point (1,0)?

1 $t = 0 \to (0,0)$ $\quad 1 - t^2 = 1 \to t = 0$

$t^3 - t = 0$

(b) For which *two* values of t does this curve go through the origin?

2 $1 - t^2 = 0 \Rightarrow t = \pm 1$

$t^3 - t = 0$

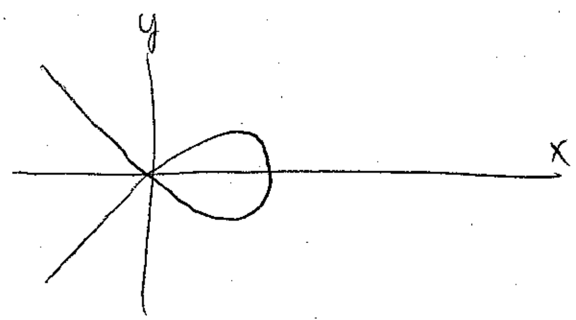
(c) What happens to the values of x and y as $t \to \infty$? As $t \to -\infty$

2 as $t \to \infty$, $x \to -\infty$ $\quad y \to \infty$

as $t \to -\infty$, $x \to -\infty$, $y \to -\infty$

(d) Sketch this curve.

2

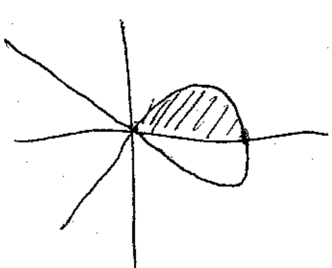


(e) For which t values does this curve have a horizontal tangent line?

horiz. tangent line: $y'(t) = 0$

2 $3t^2 - 1 = 0 \quad t = \pm \sqrt[3]{1/3}$

(f) Find the area below this curve and above the positive x-axis.



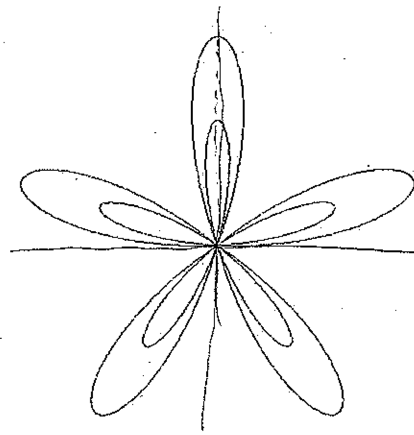
$$\int_{x=0}^{x=1} y \, dx = \int_{t=-1}^{t=0} (t^3 - t)(-2t)\, dt$$

$$= \int_{-1}^{0} -2t^4 + 2t^2 \, dt = \frac{2}{3}t^3 - \frac{2}{5}t^5 \Big|_{-1}^{0}$$

3

$$= 0 - \left(\frac{-2}{3} + \frac{2}{5}\right) = \boxed{\frac{4}{15}}$$

5. Below is the graph of $r = 1 + 4\sin(5\theta)$:



(a) What accounts for the number of leaves on this figure?

$r = 1 + 4\sin(5\theta)$

$\sin n\theta$ has n leaves for n odd, so this gives the 5 leaves.

1

(b) How long are the outer leaves? How long are the inner leaves? How do you know?

max $r = 1 + 4 = 5$ outer leaves are 5 long

min $r = 1 - 4 = -3$ so the inner leaves are 3 long.

2

we find this since $-1 \le \sin 5\theta \le 1$

6. Use parametric equations and the formula for arc length to determine the circumference of a circle of radius R.

4

$x = R\cos t$

$y = R\sin t$

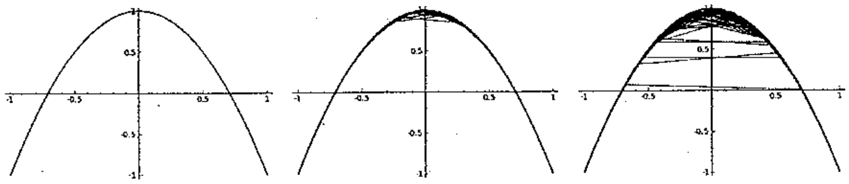
$0 \le t \le 2\pi$

$$\int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \int_0^{2\pi} \sqrt{(-R\sin t)^2 + (R\cos t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{R^2(\sin^2 t + \cos^2 t)}\, dt$$

$$= \int_0^{2\pi} R\, dt = \boxed{2\pi R}$$

7. (Bonus) See the graphs below. They represent the graph of the polar equations $x = \sin(t)$ and $y = \cos(2t)$, which we showed in class to be a parabola similar to $y = 1 - 2x^2$, but restricted and periodic. The three graphs have increasing bounds on t. Why might we get different pictures for the graphs?



(The ranges on t are $(0, 2\pi)$, $(0, 20\pi)$, and $(0, 50\pi)$).

The computer draws by connecting adjacent points into line segments. As the range increases, these points get further & further apart, making for inaccuracies in the line segments.