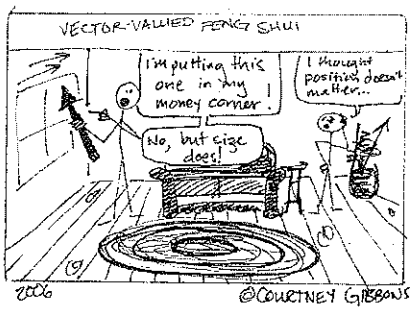
KEY

Math 225: Quiz the Third
September 23, 2011

This quiz is closed book and closed notes. Please justify all of your answers. You have 40 minutes.



1. Let $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ be vectors. For each quantity, state whether it is a vector, a scalar, or nonsense.

(a) $\mathbf{a} + (\mathbf{b} \times \mathbf{c})$

$\vec{a} + (\vec{b} \times \vec{c})$

vector + vector = vector

(b) $\mathbf{a} + (\mathbf{b} \cdot \mathbf{c})$

vector + scalar = nonsense!

6

(c) $\frac{\mathbf{a}}{|\mathbf{b}|} \cdot \frac{\mathbf{b}}{|\mathbf{a}|}$

vector · vector = scalar

(d) $\mathbf{a}^2$

nonsense → we never square a vector!

2. Consider the points $A = (2, 1, 0)$, $B = (3, 0, 4)$ and $C = (1, 2, -2)$

(a) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

$\overrightarrow{AB} = \langle 3-2, 0-1, 4-0 \rangle = \langle 1, -1, 4 \rangle$

$\overrightarrow{AC} = \langle 1-2, 2-1, -2-0 \rangle = \langle -1, 1, -2 \rangle$

1

(b) Is $\angle BAC$ acute, right, or obtuse?

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 1(-1) + (-1)\cdot 1 + 4(-2) = -10 < 0$

So $\angle BAC$ is OBTUSE.

2

(c) Find the area of $\triangle ABC$. (Hint: It is half of a certain parallelogram)

$\frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2} |\langle 2-4, -4+2, 1-1 \rangle|$

$= \frac{1}{2} |\langle -2, -2, 0 \rangle| = \frac{1}{2}\sqrt{8} = \sqrt{2}$

2

(d) Find the equation of the plane that contains $\triangle ABC$.

$\vec{n} = \langle -2, -2, 0 \rangle$

Point $= (2, 1, 0)$

plane: $-2(x-2) - 2(y-1) + 0(z-0) = 0$

$-2x + 4 - 2y + 2 = 0 \qquad -2x - 2y = 6$

or $x + y = 3$

2

(e) Find the equation of the line through points B and C.

$\vec{v} = \overrightarrow{BC} = \langle 1-3, 2-0, -2-4 \rangle = \langle -2, 2, -6 \rangle$

point: $(3, 0, 4)$

line: $\begin{cases} x = 3 - 2t \\ y = 2t \\ z = 4 - 6t \end{cases}$

2

3. (a) Rewrite the vector equation

$$\mathbf{r}(t) = \langle 1, -1, 2\rangle + t\langle 2, 3, 1\rangle$$

in parametric form.

$x = 1 + 2t$

$y = -1 + 3t$

2 $z = 2 + t$

(b) Show $\mathbf{r}(t)$ lies on the plane $5x - 3y - z = 6$.

$5(1+2t) - 3(-1+3t) - (2+t)$

$= 5 + 10t + 3 - 9t - 2 - t$

3 $= 5 + 3 - 2 = 6$ ✓ "ok."

4. Suppose that $\mathbf{a}$ and $\mathbf{b}$ are vectors such that $|\mathbf{a}| = |\mathbf{b}|$. Prove that $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$ are orthogonal.

$|\vec{a}| = |\vec{b}|$

$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = \vec{a}\cdot\vec{a} + \vec{b}\cdot\vec{a} - \vec{a}\cdot\vec{b} - \vec{b}\cdot\vec{b}$ (since $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$)

$= |\vec{a}|^2 - |\vec{b}|^2$

5 $= 0$ since $|\vec{a}| = |\vec{b}|$

so $\vec{a} + \vec{b} \perp \vec{a} - \vec{b}$.

5. Bonus: You may have 0.5 points extra credit, or 1.5 points extra credit. Note: If more than 25% of you take 1.5 points, no one gets any extra credit.

I DON'T GET A VOTE.